

Foundations of Materials Science and Engineering

Lecture Note 10

June 10, 2013

Kwang Kim

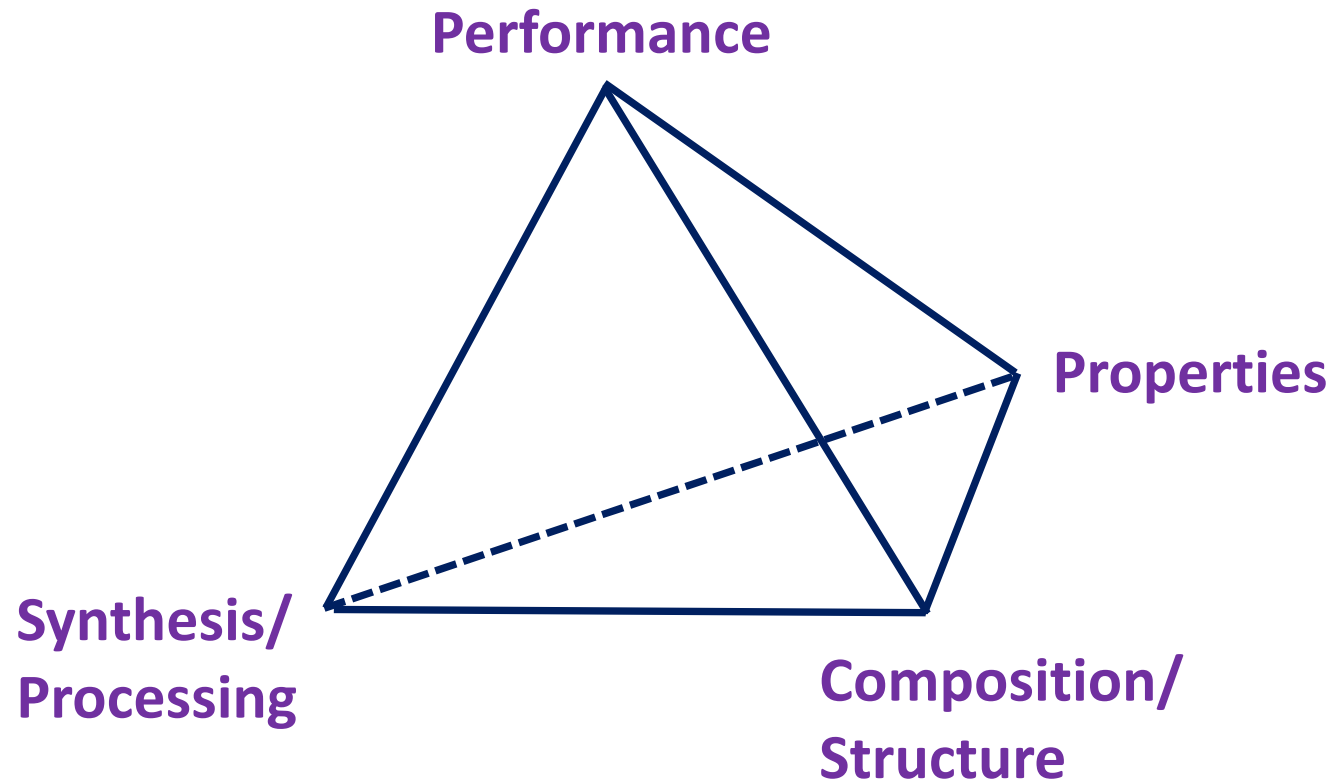
Yonsei University

kbkim@yonsei.ac.kr

39	8	7	34	53
Y	O	N	Se	I
88.91	16.00	14.01	78.96	126.9

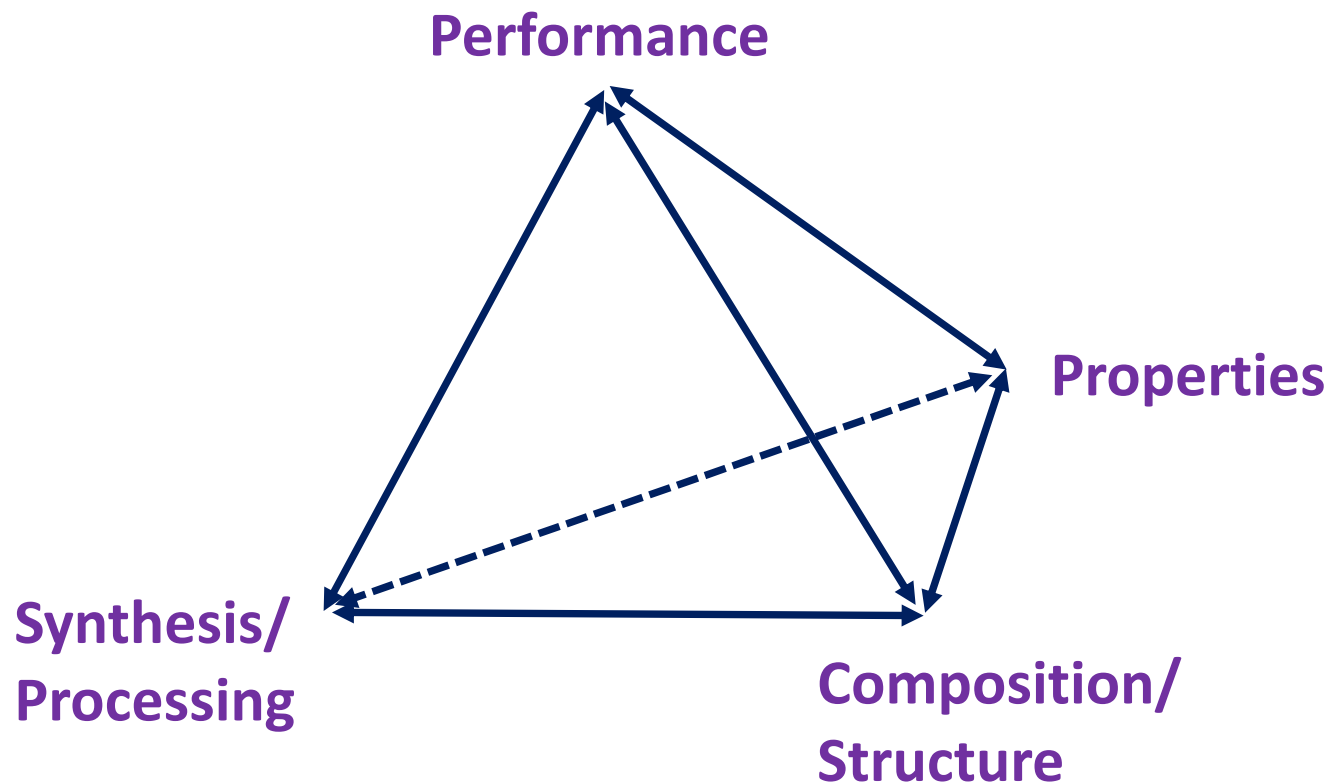
Material Science and Engineering (MSE)

"Tetrahedron of Materials Science and Engineering"



Material Science and Engineering (MSE)

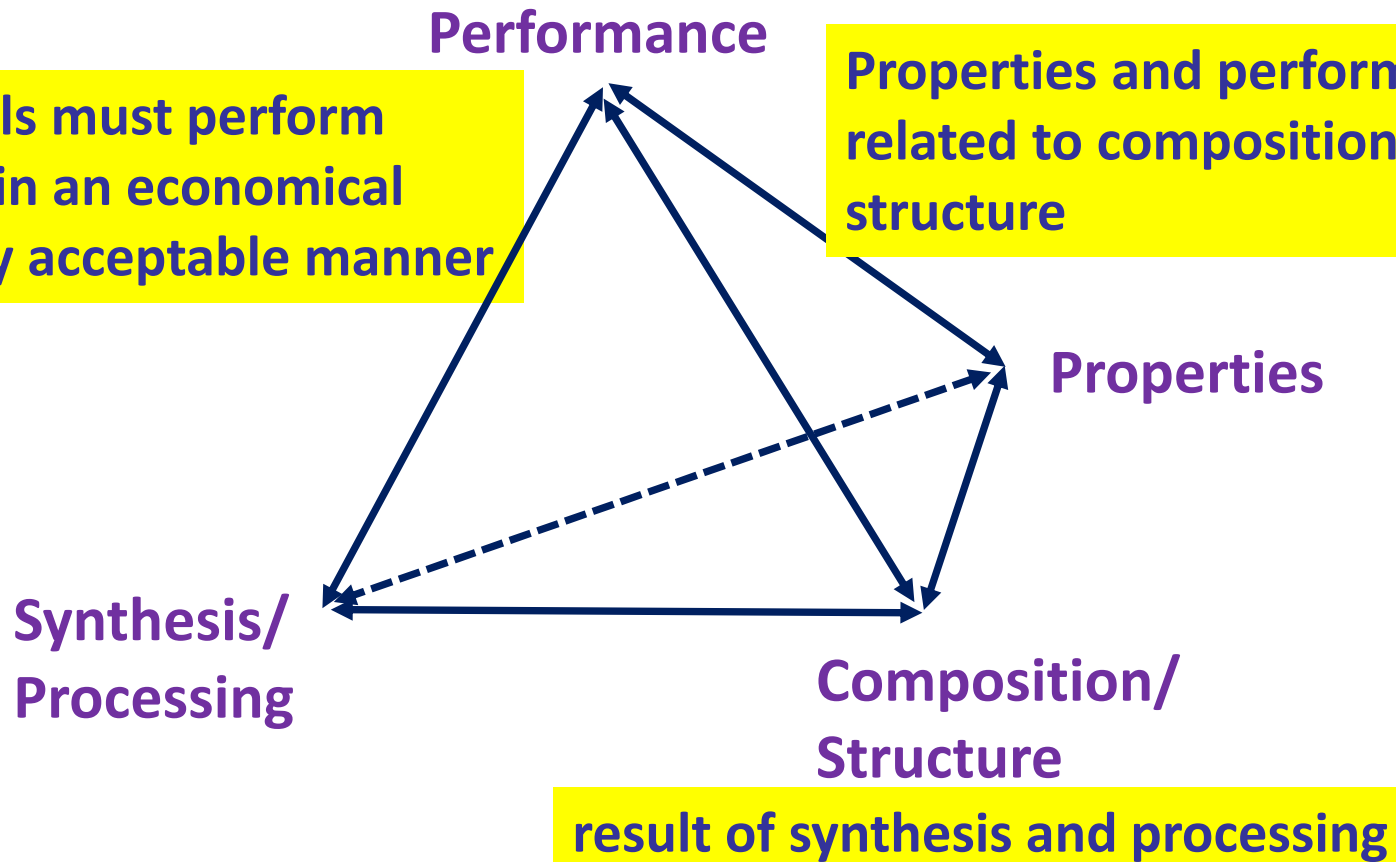
Four elements of materials and strong interrelationship among them define a field of Materials Science and Engineering. Materials Science and Engineering rooted in the classical description of physics and chemistry



Material Science and Engineering (MSE)

Final materials must perform a given task in an economical and societally acceptable manner

Properties and performance : related to composition and structure



Material Selection

1. Pick **Application and Determine required Properties**

Properties: mechanical, electrical, thermal, magnetic, optical

2. **Properties : Identify candidate Material(s)**

Material: structure, composition

3. **Material : Identify required Processing**

Processing: changes structure and overall shape

ex: casting, sintering, vapor deposition, forming, joining,

Periodic Table

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display

Atomic number increases
Atomic radius decreases
Ionization energy increases

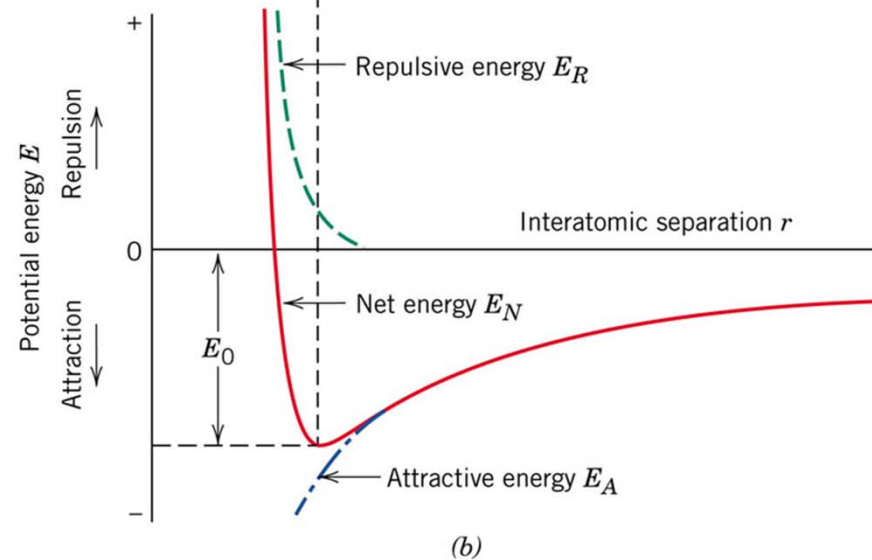
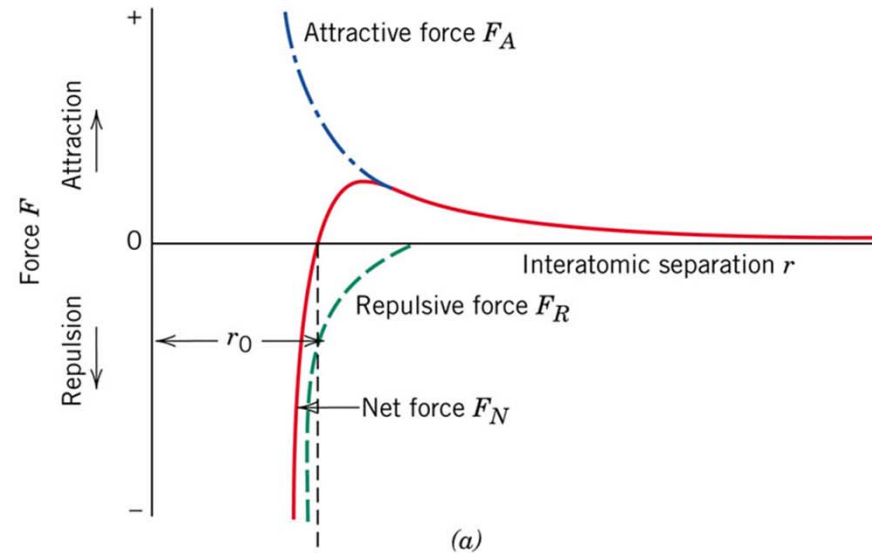
Atomic number increases
Atomic radius increases
Ionization energy decreases

Main-Group Elements (s block)		Transition Elements (d block)										Main-Group Elements (p block)						
1A (1)	2A (2)											3A (13)	4A (14)	5A (15)	6A (16)	7A (17)	8A (18)	
ns^1	ns^2											ns^2np^1	ns^2np^2	ns^2np^3	ns^2np^4	ns^2np^5	ns^2np^6	
1 H $1s^1$																		2 He $1s^2$
2 Li $2s^1$	Be $2s^2$	Transition Elements (d block)										B $2s^22p^1$	C $2s^22p^2$	N $2s^22p^3$	O $2s^22p^4$	F $2s^22p^5$	Ne $2s^22p^6$	
3 Na $3s^1$	Mg $3s^2$	3B (3)	4B (4)	5B (5)	6B (6)	7B (7)	8B (8) (9) (10)			1B (11)	2B (12)	Al $3s^23p^1$	Si $3s^23p^2$	P $3s^23p^3$	S $3s^23p^4$	Cl $3s^23p^5$	Ar $3s^23p^6$	
4 K $4s^1$	Ca $4s^2$	Sc $4s^23d^1$	Ti $4s^23d^2$	V $4s^23d^3$	Cr $4s^13d^5$	Mn $4s^23d^5$	Fe $4s^23d^6$	Co $4s^23d^7$	Ni $4s^23d^8$	Cu $4s^13d^{10}$	Zn $4s^23d^{10}$	Ga $4s^24p^1$	Ge $4s^24p^2$	As $4s^24p^3$	Se $4s^24p^4$	Br $4s^24p^5$	Kr $4s^24p^6$	
5 Rb $5s^1$	Sr $5s^2$	Y $5s^24d^1$	Zr $5s^24d^2$	Nb $5s^14d^4$	Mo $5s^24d^5$	Tc $5s^24d^5$	Ru $5s^14d^7$	Rh $5s^14d^8$	Pd $4d^{10}$	Ag $5s^14d^{10}$	Cd $5s^24d^{10}$	In $5s^25p^1$	Sn $5s^25p^2$	Sb $5s^25p^3$	Te $5s^25p^4$	I $5s^25p^5$	Xe $5s^25p^6$	
6 Cs $6s^1$	Ba $6s^2$	La* $6s^25d^1$	Hf $6s^25d^2$	Ta $6s^25d^3$	W $6s^25d^4$	Re $6s^25d^5$	Os $6s^25d^6$	Ir $6s^25d^7$	Pt $6s^15d^9$	Au $6s^15d^{10}$	Hg $6s^25d^{10}$	Tl $6s^26p^1$	Pb $6s^26p^2$	Bi $6s^26p^3$	Po $6s^26p^4$	At $6s^26p^5$	Rn $6s^26p^6$	
7 Fr $7s^1$	Ra $7s^2$	Ac** $7s^26d^1$	Rf $7s^26d^2$	Db $7s^26d^3$	Sg $7s^26d^4$	Bh $7s^26d^5$	Hs $7s^26d^6$	Mt $7s^26d^7$	Ds $7s^26d^8$	Rg $7s^26d^9$	$7s^26d^{10}$	$7s^27p^1$	$7s^27p^2$	$7s^27p^3$	$7s^27p^4$			
Inner Transition Elements (f block)																		
6	*Lanthanides	Ce $6s^24f^15d^1$	Pr $6s^24f^3$	Nd $6s^24f^4$	Pm $6s^24f^5$	Sm $6s^24f^6$	Eu $6s^24f^7$	Gd $6s^24f^75d^1$	Tb $6s^24f^9$	Dy $6s^24f^{10}$	Ho $6s^24f^{11}$	Er $6s^24f^{12}$	Tm $6s^24f^{13}$	Yb $6s^24f^{14}$	Lu $6s^24f^{14}5d^1$			
7	**Actinides	Th $7s^26d^2$	Pa $7s^25f^26d^1$	U $7s^25f^36d^1$	Np $7s^25f^46d^1$	Pu $7s^25f^6$	Am $7s^25f^7$	Cm $7s^25f^76d^1$	Bk $7s^25f^9$	Cf $7s^25f^{10}$	Es $7s^25f^{11}$	Fm $7s^25f^{12}$	Md $7s^25f^{13}$	No $7s^25f^{14}$	Lr $7s^25f^{14}6d^1$			

Primary Bonds

- Bonding with other atoms, the potential energy of each bonding atom is lowered resulting in a more stable state.
- Three primary bonding combinations : 1) metal-nonmetal, 2) nonmetal-nonmetal, and 3) metal-metal.
- **Ionic bonds** :- Strong atomic bonds due to transfer of **electrons**
- **Covalent bonds** :- Large interactive force due to sharing of **electrons**
- **Metallic bonds** :- Non-directional bonds formed by sharing of **electrons**
- **Permanent Dipole bonds** :- Weak intermolecular bonds due to attraction between the ends of permanent **dipoles**.
- **Fluctuating Dipole bonds** :- Very weak electric **dipole** bonds due to asymmetric distribution of electron densities.

Intermolecular Forces and Potentials

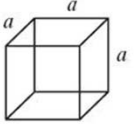
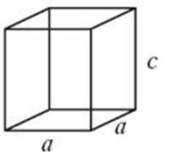
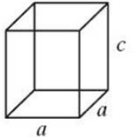
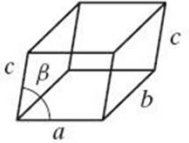
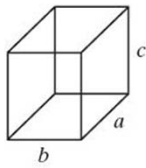
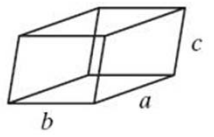



$$F_{net} = \frac{Z_1 Z_2 e^2}{(4\pi \epsilon_0 a^2)} - \frac{nb}{a^{n+1}}$$

$$F = - \partial E / \partial r$$

7 Crystal Systems

TABLE 3.1

The Seven Crystal Systems					
System	Axial lengths and angles ^a	Unit cell geometry			
Cubic	$a = b = c, \alpha = \beta = \gamma = 90^\circ$		Hexagonal	$a = b \neq c, \alpha = \beta = 90^\circ, \gamma = 120^\circ$	
Tetragonal	$a = b \neq c, \alpha = \beta = \gamma = 90^\circ$		Monoclinic	$a \neq b \neq c, \alpha = \gamma = 90^\circ \neq \beta$	
Orthorhombic	$a \neq b \neq c, \alpha = \beta = \gamma = 90^\circ$		Triclinic	$a \neq b \neq c, \alpha \neq \beta \neq \gamma \neq 90^\circ$	
Rhombohedral	$a = b = c, \alpha = \beta = \gamma \neq 90^\circ$				

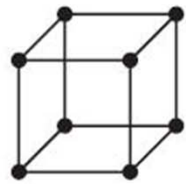
The lattice parameters a , b , and c are unit cell edge lengths.

The lattice parameters α , β , and γ are angles between adjacent unit-cell axes, where α is the angle viewed along the a axis (i.e., the angle between the b and c axes). The inequality sign (\neq) means that equality is not required.

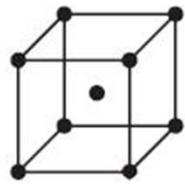
14 Bravais Lattices

TABLE 3.2

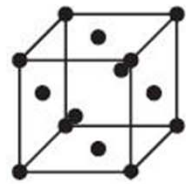
The 14 Crystal (Bravais) Lattices



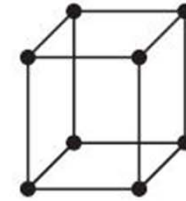
Simple cubic



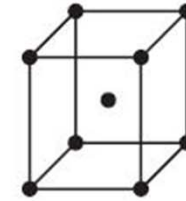
Body-centered cubic



Face-centered cubic



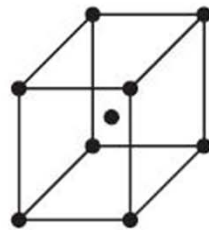
Simple tetragonal



Body-centered tetragonal



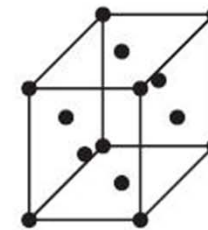
Simple orthorhombic



Body-centered orthorhombic



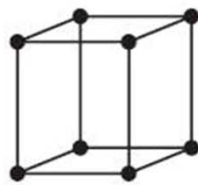
Base-centered orthorhombic



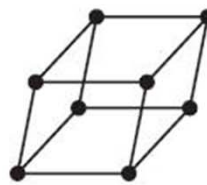
Face-centered orthorhombic



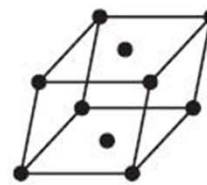
Rhombohedral



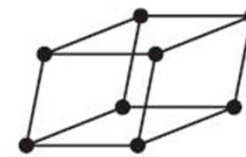
Hexagonal



Simple monoclinic



Base-centered monoclinic



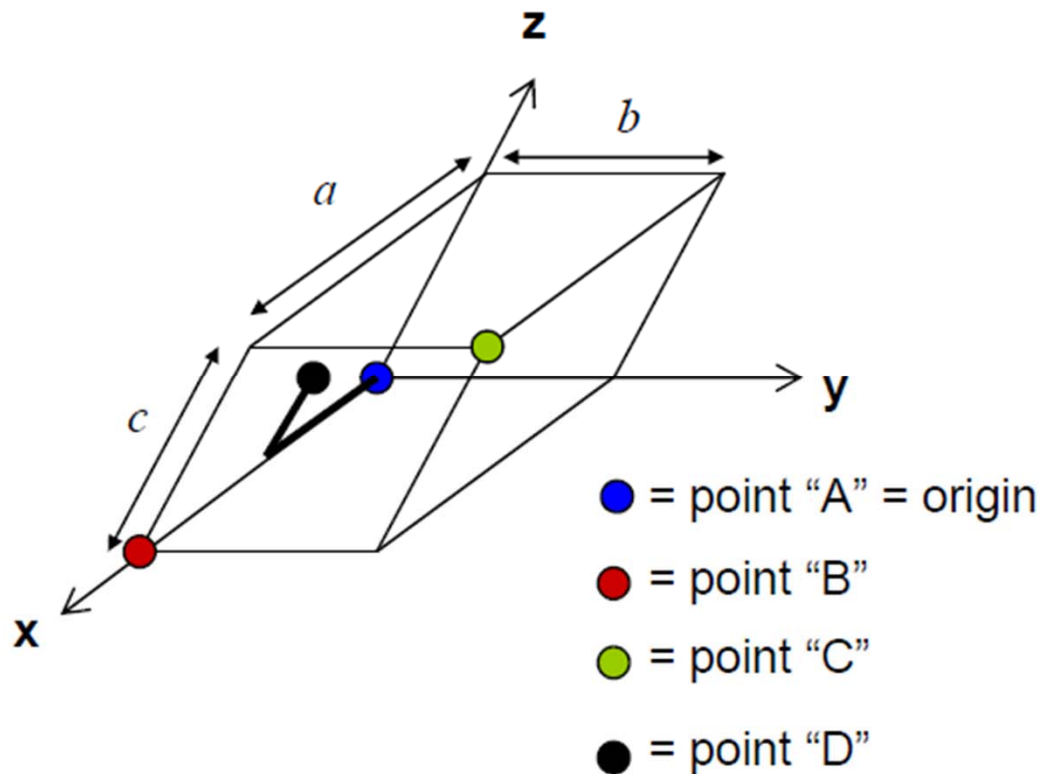
Triclinic

Atom Positions in Cubic Unit Cells

Point Coordinates

to define a point within a unit cell....

Essentially same as Cartesian coordinates except values of x , y , and z are expressed as fractions of the magnitude of unit vector(s) (and x , y , and z not necessarily orthogonal).



	pt. coord.		
	$x (a)$	$y (b)$	$z (c)$
A	0	0	0
B	1	0	0
C	1	1	1
D	1/2	0	1/2

Crystallographic Directions

To define a vector:

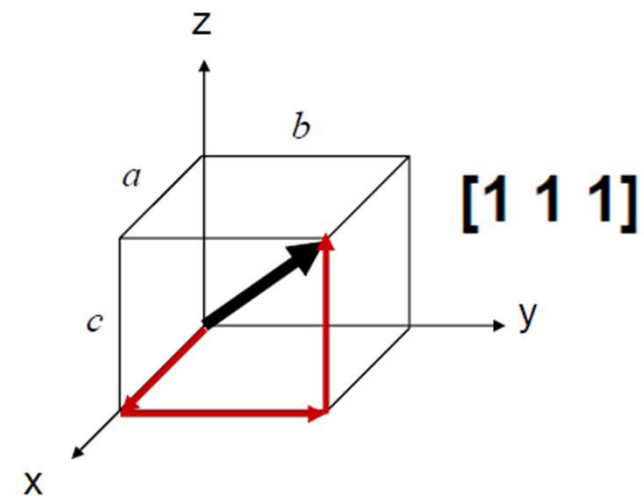
1. Start at the origin.
2. Determine length of vector projection in each of 3 axes in units (or fractions) of a , b , and c .
3. Multiply or divide by a common factor to reduce the lengths to the smallest integer values.
4. Enclose in square brackets: $[u\ v\ w]$ where u , v , and w are integers.

Along unit vectors: a b c

Note: in any of the 3 directions there are both directions.

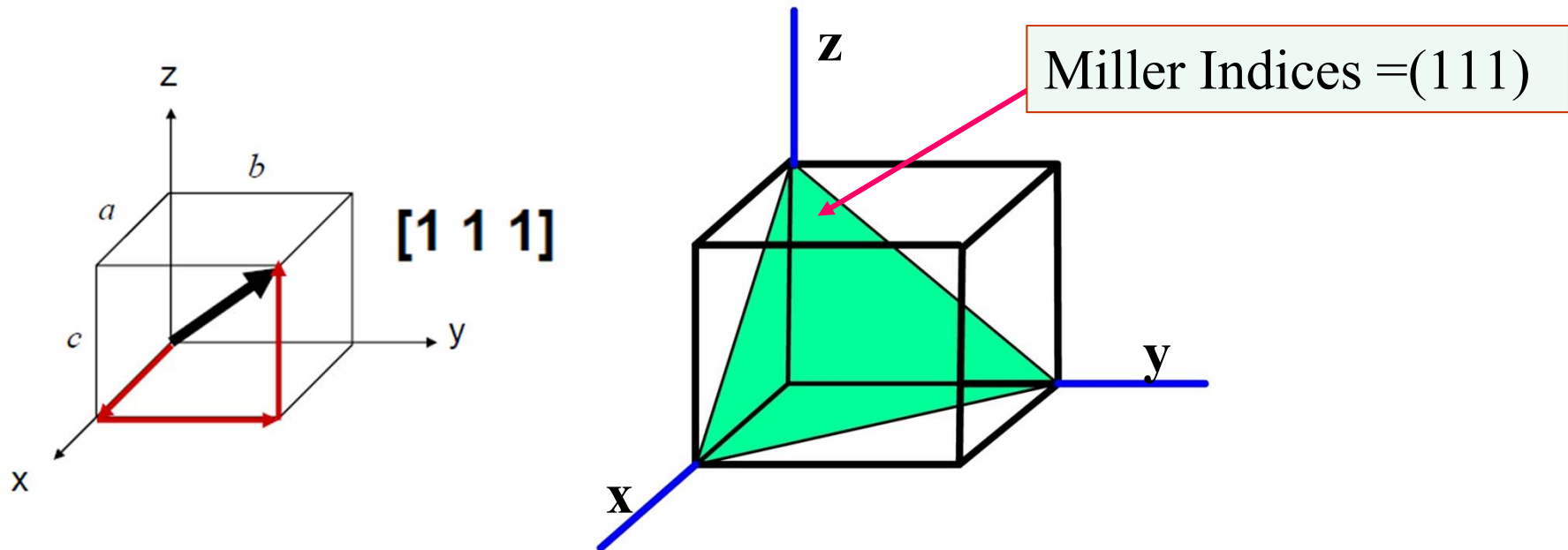
Negative directions are denoted with a "bar" c

e.g. $[1\ \bar{1}\ 1]$



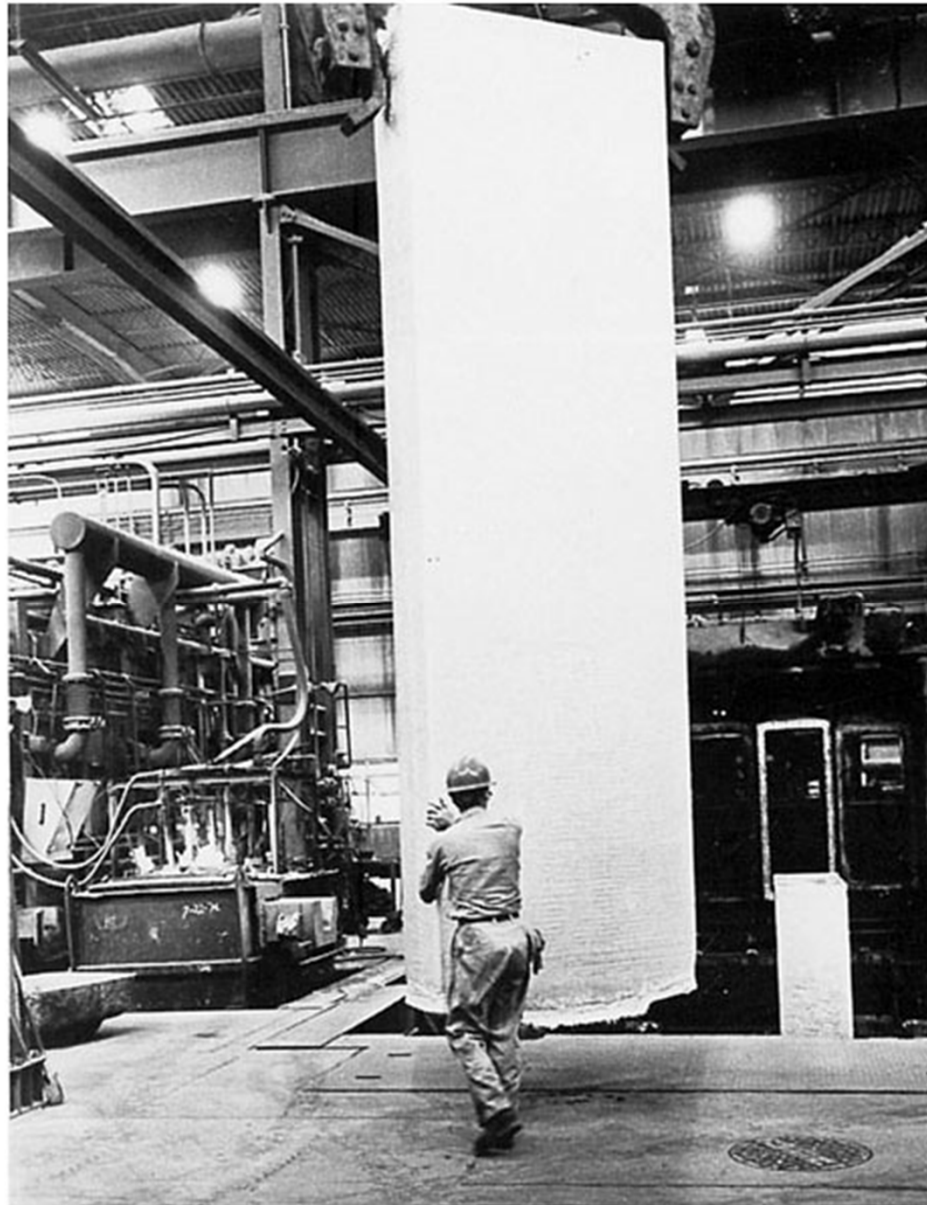
Miller Index

- **Miller indices are used to specify directions and planes in lattices or in crystals.**
- **The number of indices will match with the dimension of the lattice or the crystal.**



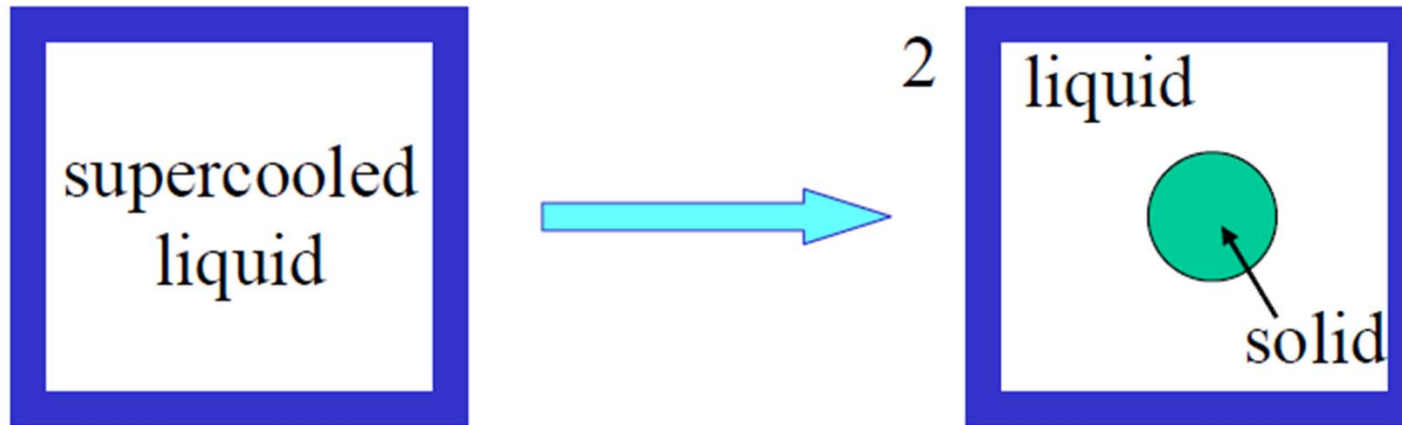
Solidification of Metals

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



Courtesy of Reynolds Metals Co.

Energies Involved in Homogeneous Nucleation



Is the transition from undercooled liquid to a solid spherical particle in the liquid a spontaneous one?

That is, is the Gibbs free energy decreases?

The formation of a solid nucleus leads to a Gibbs free energy change of ΔG

Energies Involved in Homogeneous Nucleation

The formation of a solid nucleus leads to a Gibbs free energy change of ΔG

$$\Delta G = G_2 - G_1 = \underbrace{-V_S (G_V^L - G_V^S)}_{\substack{\uparrow \\ \text{negative below } T_m}} + \underbrace{A \gamma^{SL}}_{\substack{\uparrow \\ \text{always} \\ \text{positive}}}$$

V_S is the volume of the solid sphere

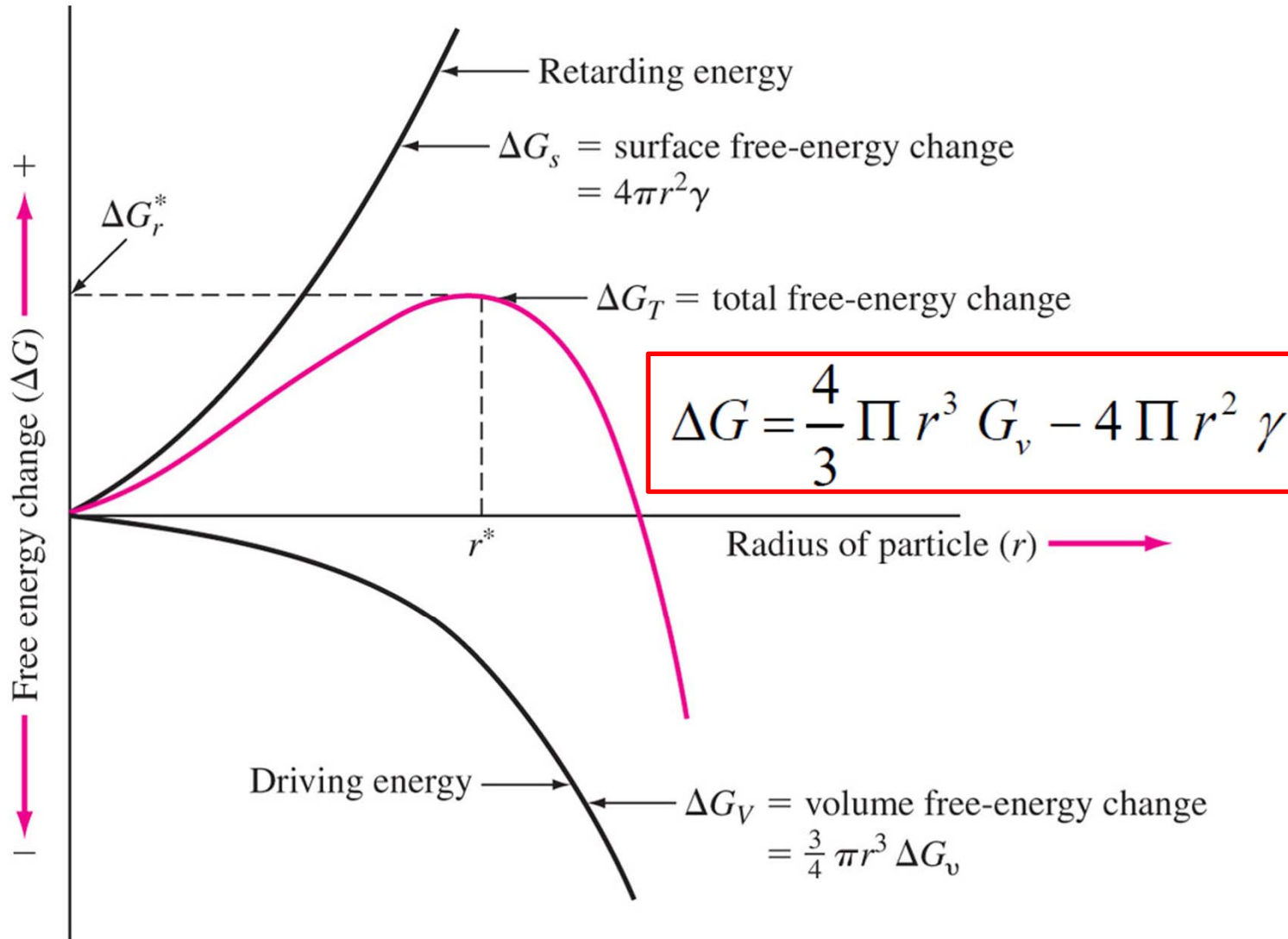
A^{SL} is the solid/liquid interfacial area

γ^{SL} is the solid/liquid interfacial energy

$\Delta G_V = G_V^L - G_V^S$ is the volume free energy difference

Energies involved in homogenous nucleation

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display



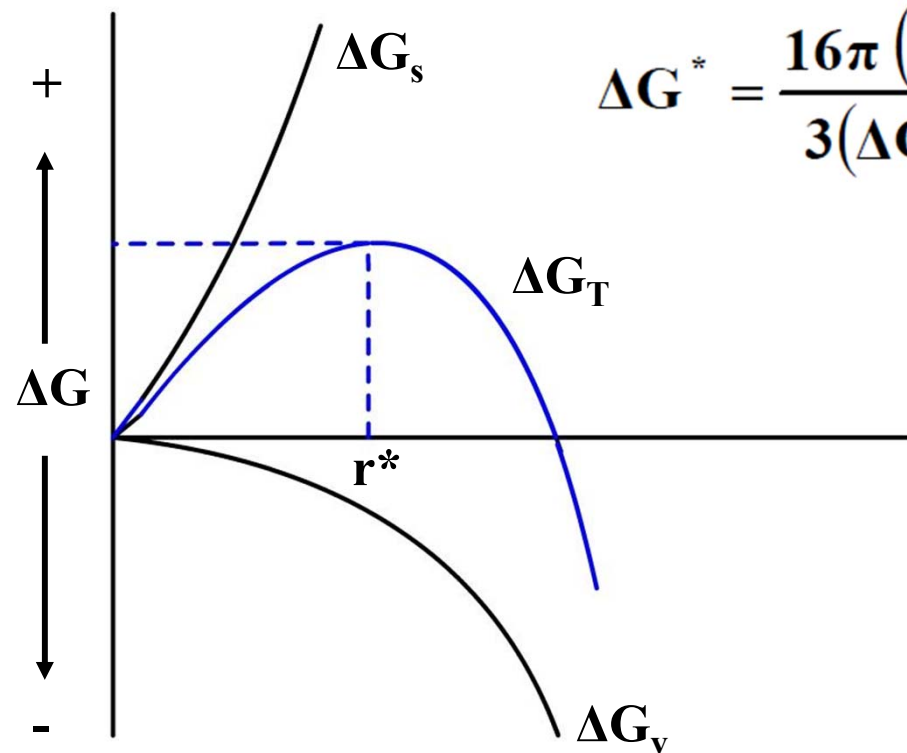
Total Free Energy

- Total free energy* is given by

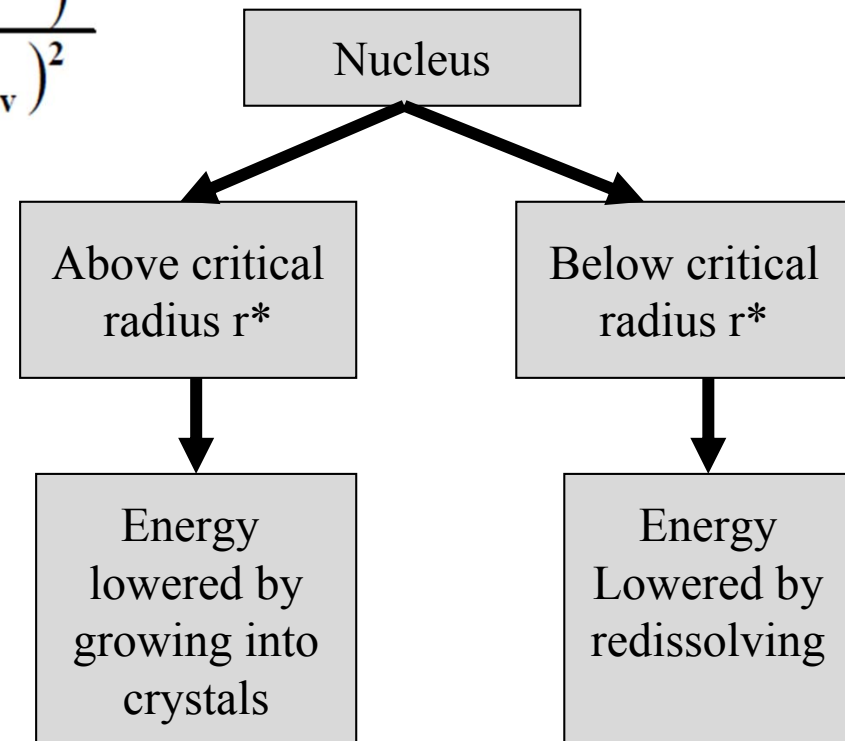
$$\Delta G = \frac{4}{3} \Pi r^3 G_v - 4 \Pi r^2 \gamma$$

Since when $r=r^*$, $d(\Delta G_T)/dr = 0$

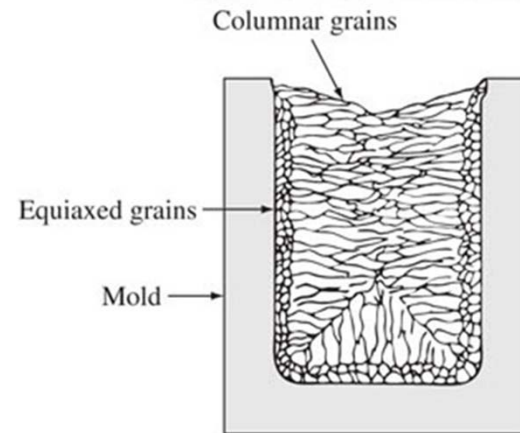
$$r^* = -\frac{2\gamma}{\Delta G_v}$$



$$\Delta G^* = \frac{16\pi (\gamma^{SL})^3}{3(\Delta G_v)^2}$$



Heterogeneous Nucleation



Heterogeneous nucleation occurs much more often than homogeneous nucleation. Heterogeneous nucleation applies to the phase transformation between any two phases of gas, liquid, or solid, typically for example, condensation of gas/vapor, solidification from liquid, bubble formation from liquid, etc.

Heterogeneous nucleation forms at preferential sites such as phase boundaries, surfaces (of container, bottles, etc.) or impurities like dust. At such preferential sites, the effective surface energy is lower, thus diminishes the free energy barrier and facilitating nucleation.

Defects in crystalline solids

1). Point Defect

- ①. **Impurity** : In an alloy some unwanted elements may be existed.
- ②. **Vacancy** : Missing atom from the lattice point.

2). Line Defect

Dislocation : Edge, screw and mixed dislocations

3). Planar Defect

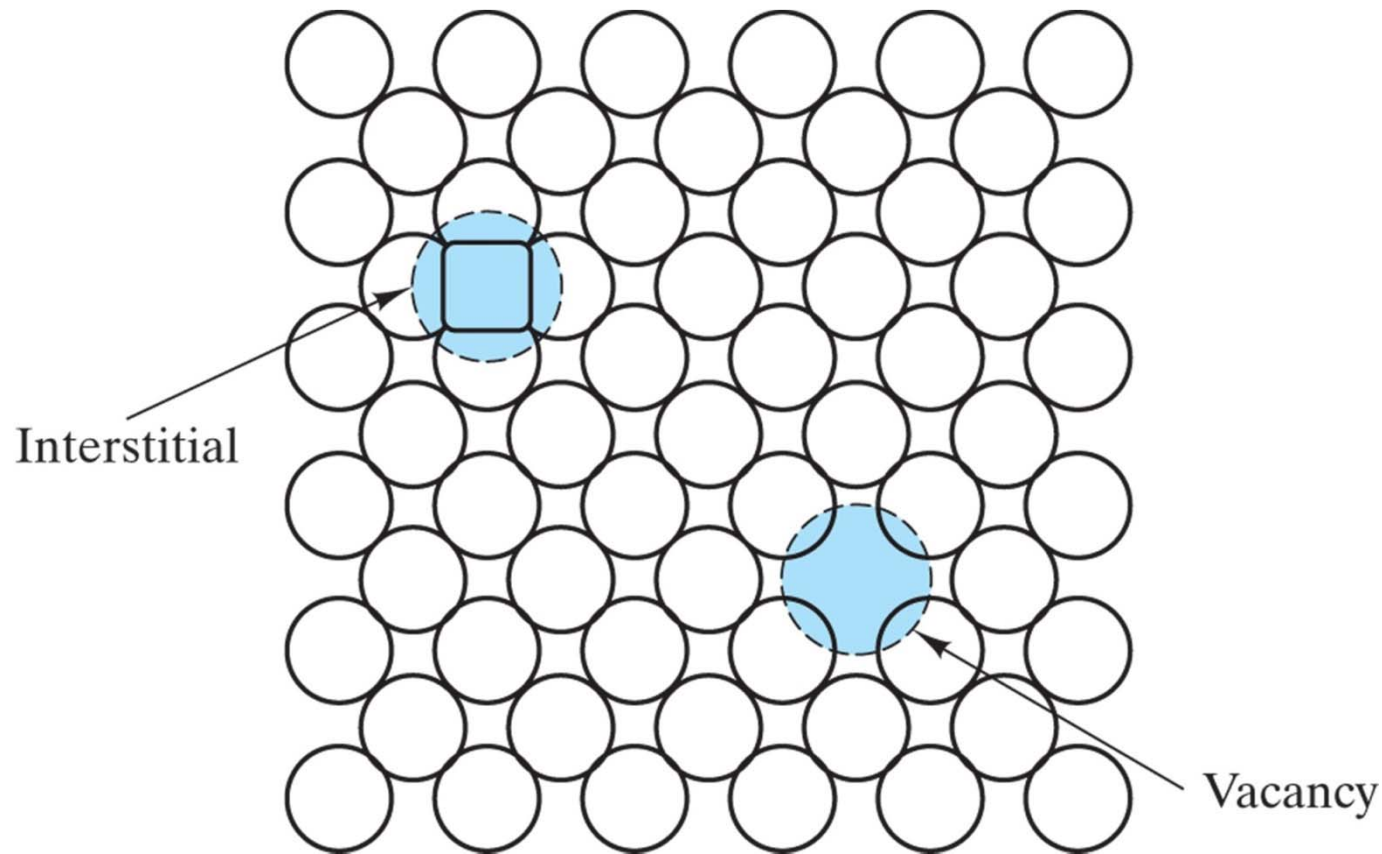
- ①. **Grain boundary**
- ②. **Twin boundary**
- ③. **Anti-phase domain boundary**

4). Volumetric Defect

Void, Cracks, Shrinkage, inclusion, etc.

Point Defects

Two common point defects in metal or elemental semiconductor structures are the vacancy and the interstitial.



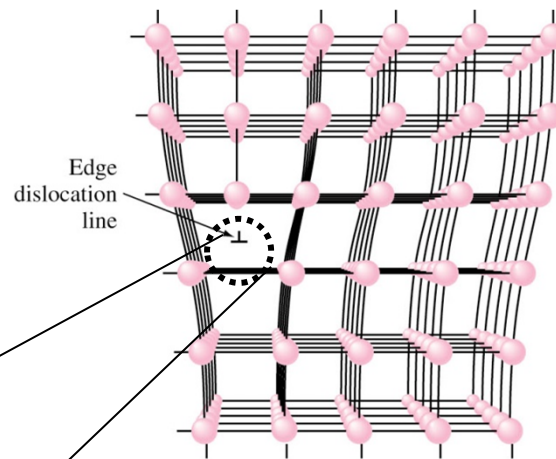
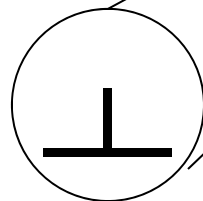
Edge Dislocation

- Created by insertion of extra half planes of atoms.

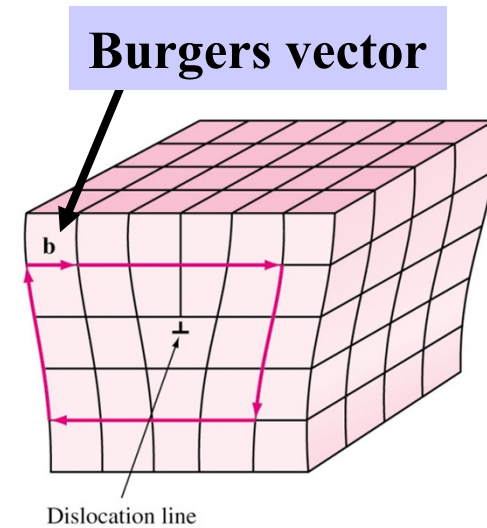
-   Positive edge dislocation

-   Negative edge dislocation

- Burgers vector
Shows displacement of atoms (slip).



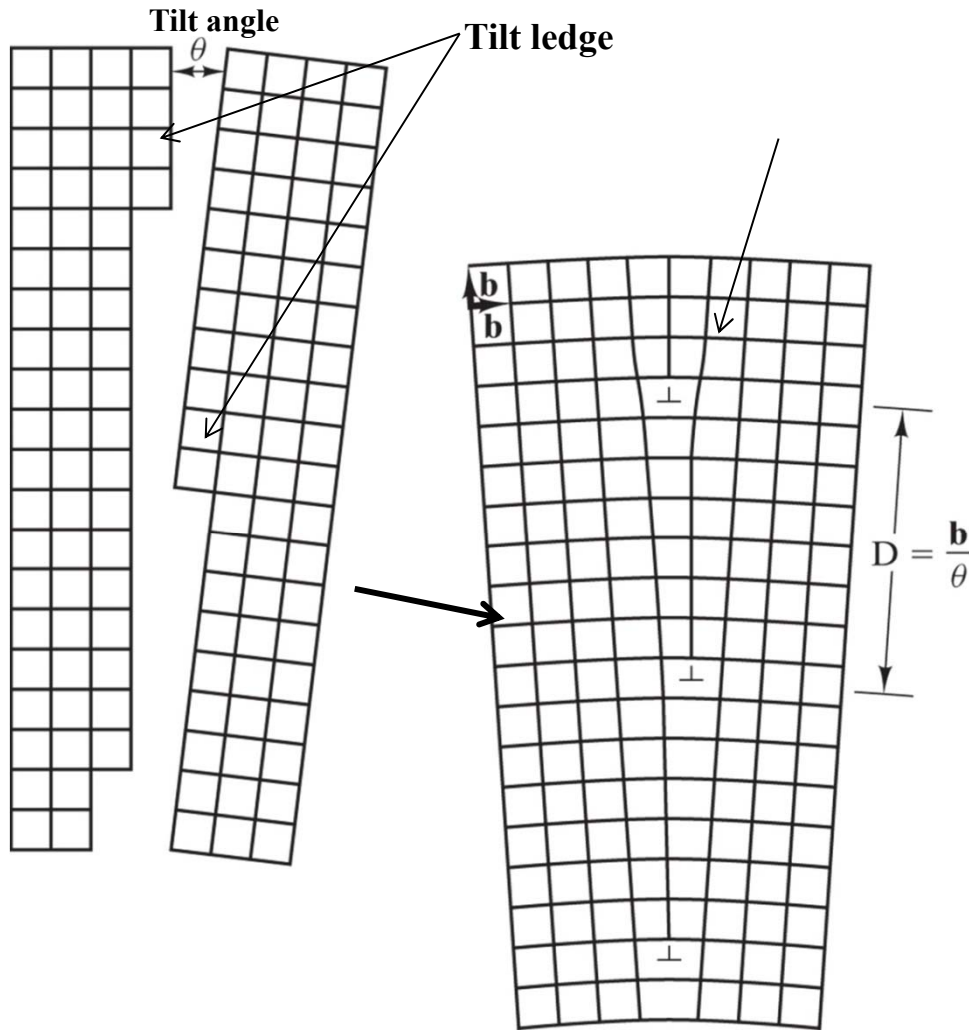
(a)



(b)

Burger's vector is perpendicular to the edge dislocation line and same as the moving direction.

Grain Boundaries



This is termed a tilt boundary because it is formed when two adjacent crystalline grains are tilted relative to each other by a few degrees (θ). The resulting structure is equivalent to isolated edge dislocations separated by the distance b/θ , where b is the length of the Burgers vector, b .

Stress and Strain in Metals

Elastic deformation: Metal returns to its original dimension after tensile force is removed.

Plastic deformation: Metal is deformed to such an extent such that it cannot return to its original dimension

Slip Mechanism

- During shear, atoms do not slide over each other.
- The slip occurs due to movement of dislocations.

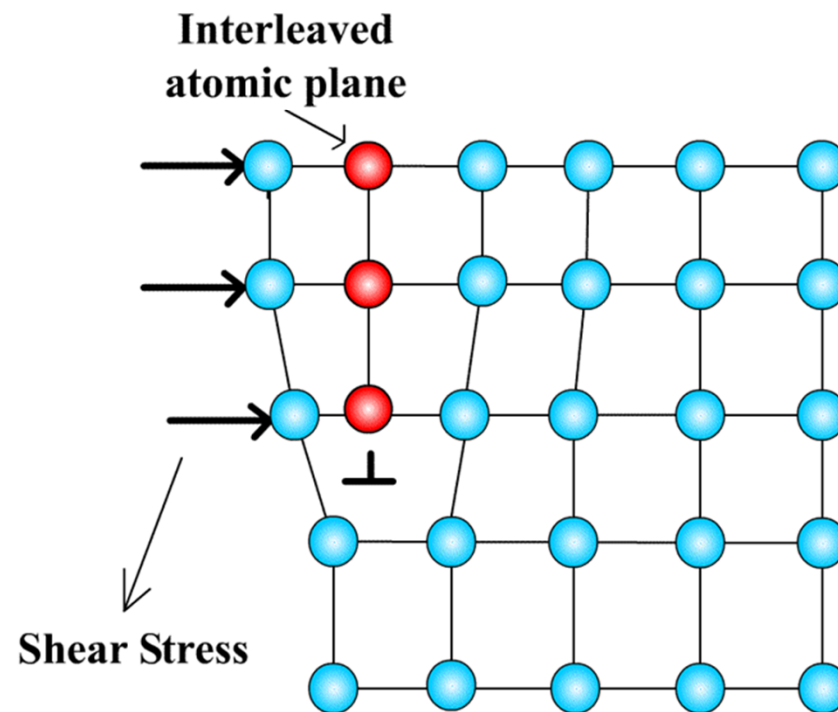
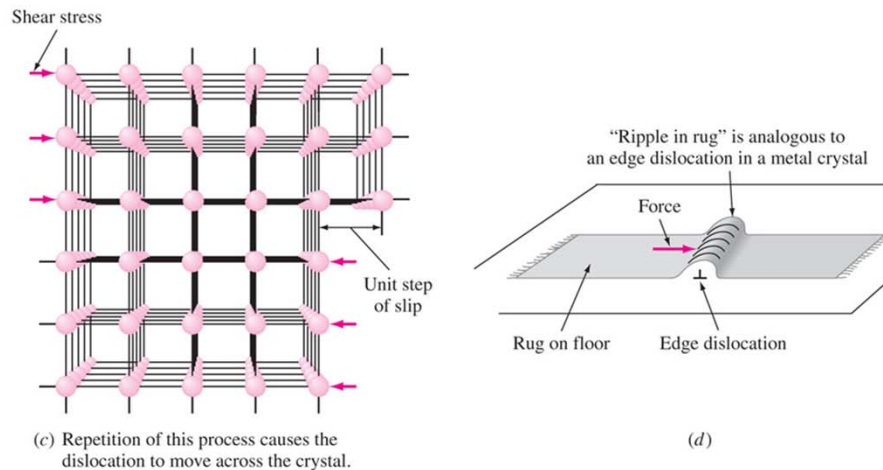
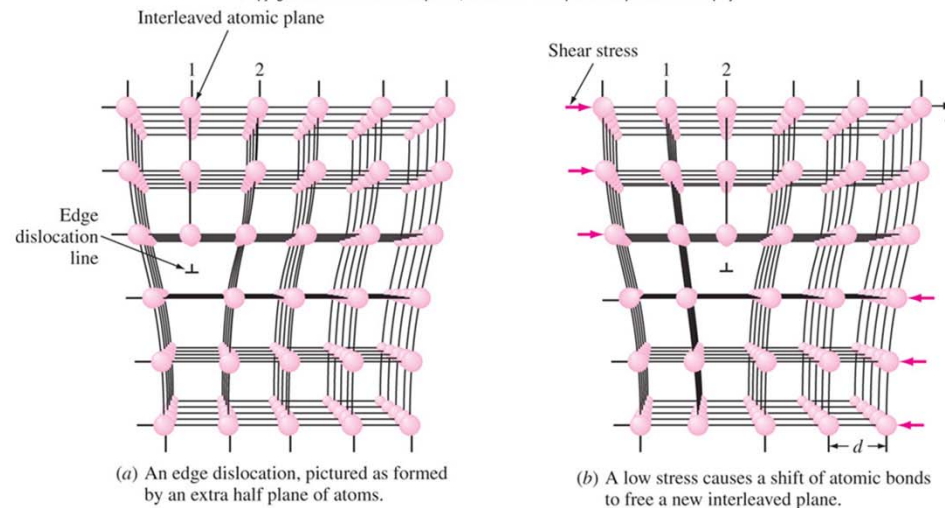


Figure 5.32

Slip Mechanism

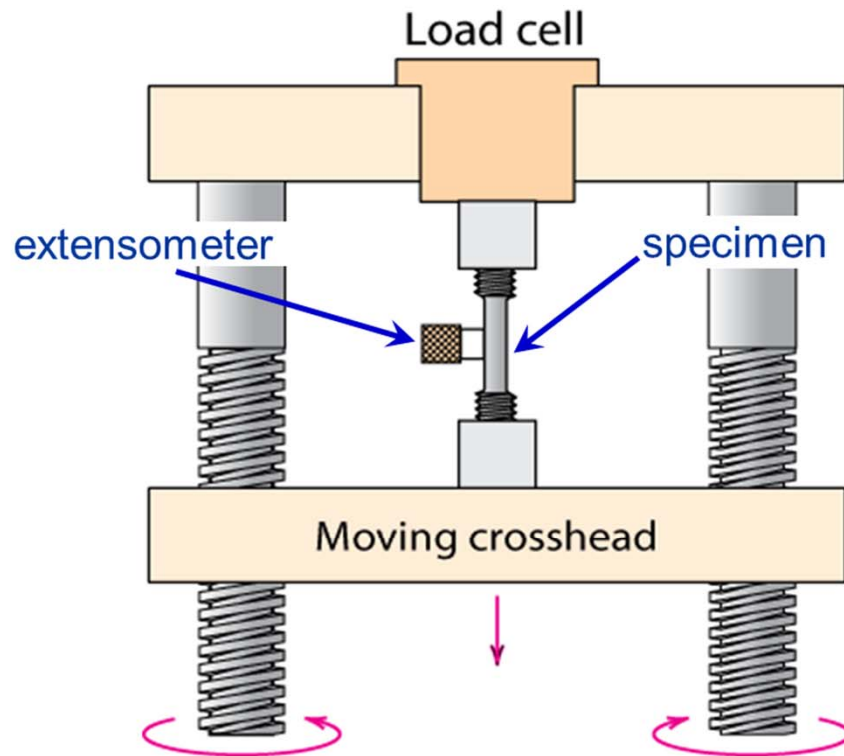
- During shear, atoms do not slide over each other.
- The slip occurs due to movement of dislocations.

Copyright © The McGraw-Hill Companies, Inc. Permission required for reproduction or display

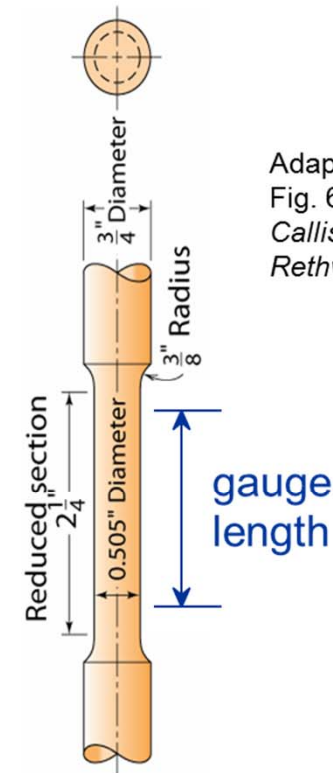


Stress-Strain Testing

- Typical tensile test machine



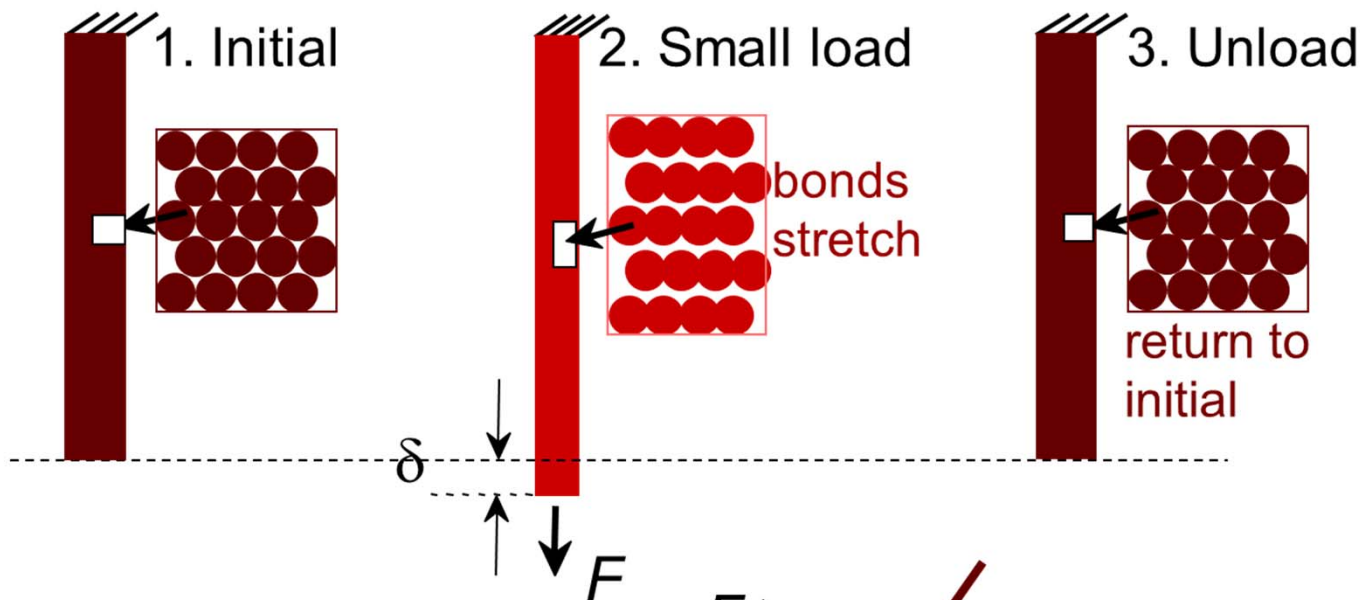
- Typical tensile specimen



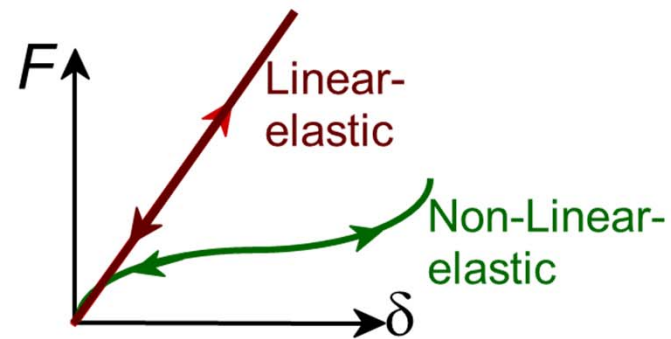
Adapted from
Fig. 6.2,
Callister &
Rethwisch 8e.

Adapted from Fig. 6.3, Callister & Rethwisch 8e. (Fig. 6.3 is taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

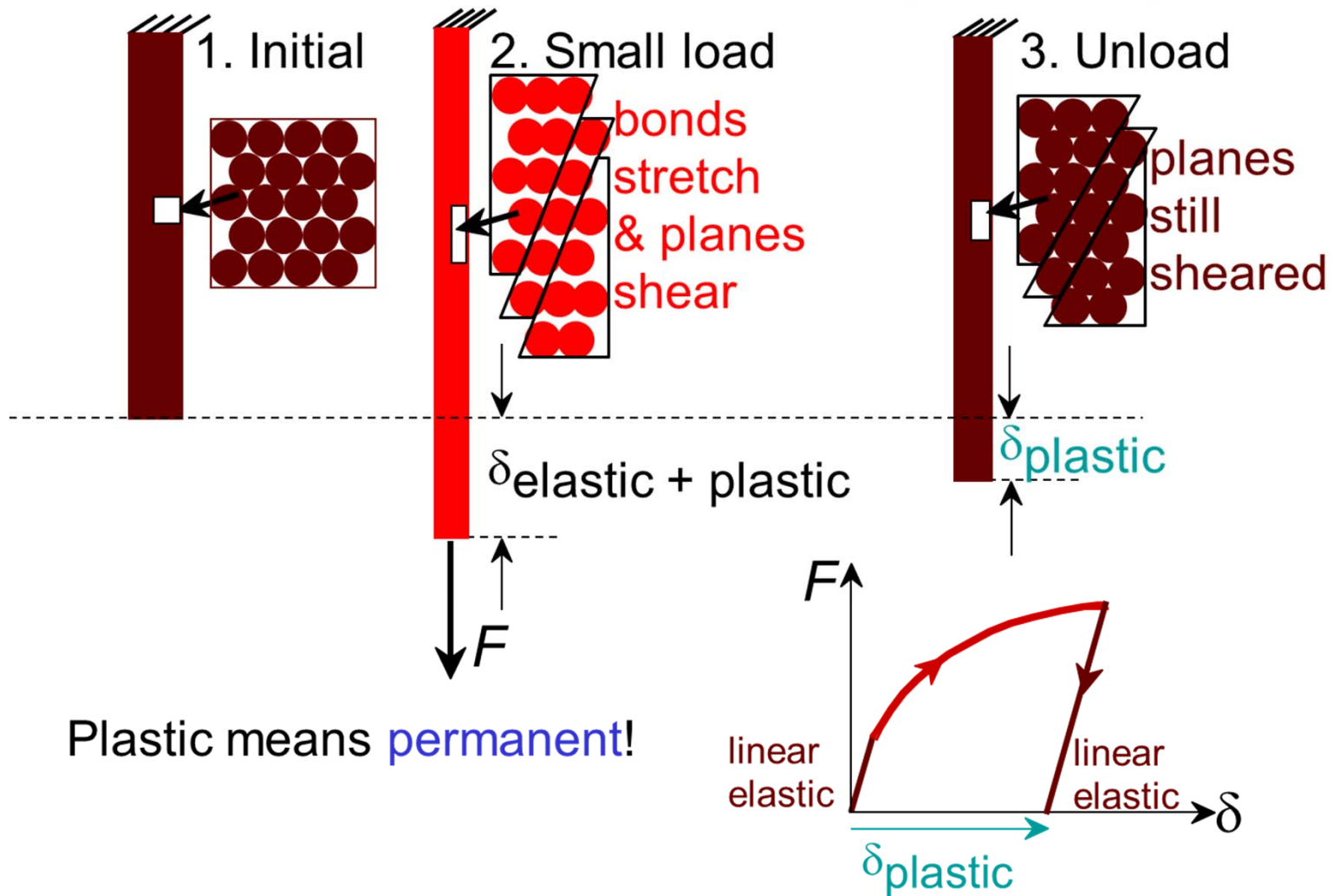
Elastic Deformation



Elastic means **reversible!**



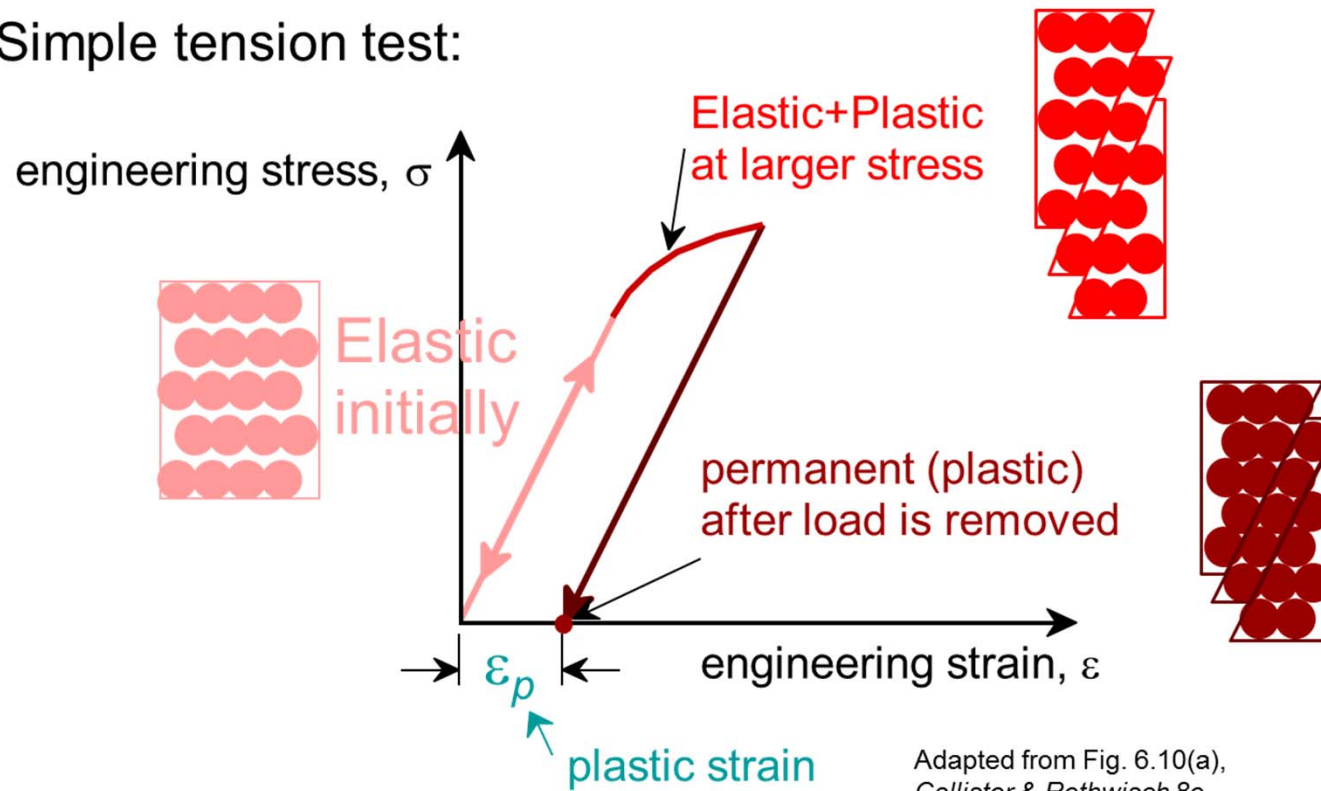
Plastic Deformation (Metals)



Plastic (Permanent) Deformation

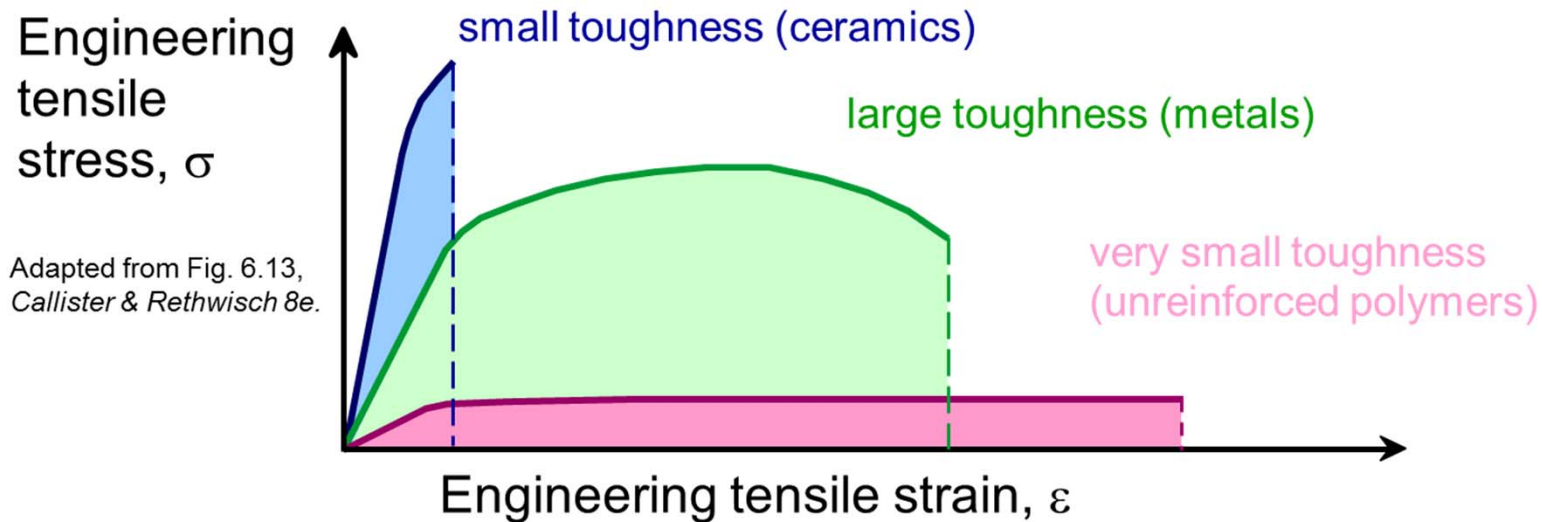
(at lower temperatures, i.e. $T < T_{melt}/3$)

- Simple tension test:



Toughness

- Energy to break a unit volume of material
- Approximate by the area under the stress-strain curve.



Brittle fracture: elastic energy

Ductile fracture: elastic + plastic energy

Ionic and Covalent Bonding in Simple Ceramics

Mixture of Ionic and Covalent Types.
Depends on electronegativity difference.

Table 11.2 Percent ionic and covalent bonding in some ceramic compounds

Ceramic compound	Bonding atoms	Electronegativity difference	% ionic character	% covalent character
Zirconium dioxide, ZrO_2	Zr-O	2.3	73	27
Magnesium oxide, MgO	Mg-O	2.2	69	31
Aluminum oxide, Al_2O_3	Al-O	2.0	63	37
Silicon dioxide, SiO_2	Si-O	1.7	51	49
Silicon nitride, Si_3N_4	Si-N	1.3	34.5	65.5
Silicon carbide, SiC	Si-C	0.7	11	89

Ceramic Crystal Structures

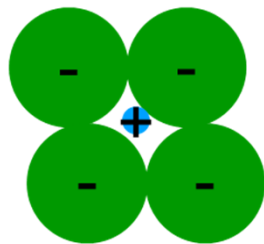
Oxide structures

- Oxygen anions (–ve ions) larger than metal cations (+ve ions)
- Close packed oxygen in a lattice (usually FCC)
- Cations fit into interstitial sites among oxygen ions

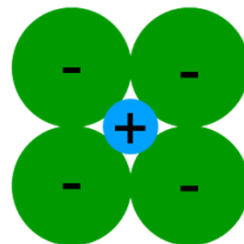
Factors that Determine Crystal Structure

1. Relative sizes of ions – Formation of stable structures:

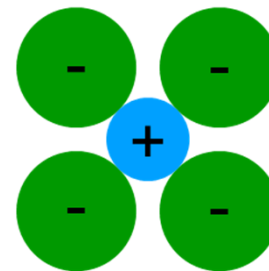
--maximize the # of oppositely charged ion neighbors.



unstable



stable



stable

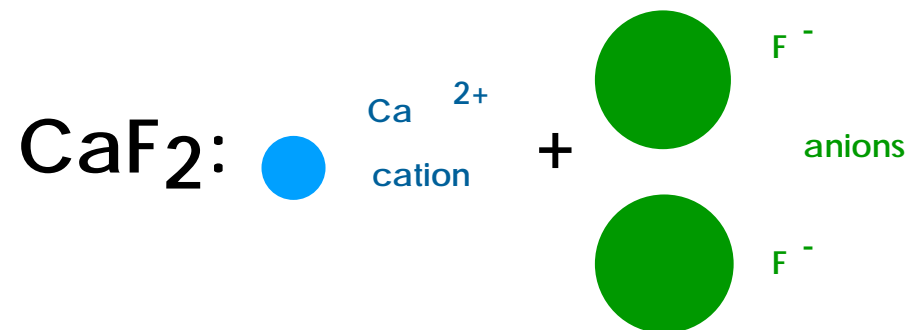
Adapted from Fig. 12.1,
Callister & Rethwisch 8e.

2. Maintenance of

Charge Neutrality :

--Net charge in ceramic
should be zero.

--Reflected in chemical
formula:



Simple Ionic Arrangements

Packing of Ions depends upon Relative size of ions.

Need to balance electron charges.

If the anion does not touch the cation,
then the arrangement is unstable.

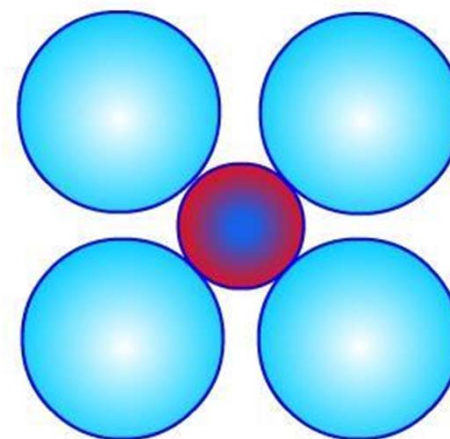
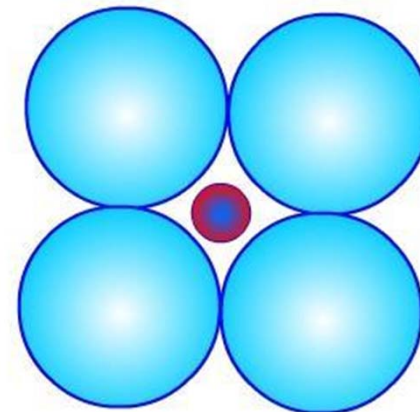
$$\text{Radius ratio} = r_{\text{cation}}/r_{\text{anion}}$$

Critical radius ratio for stability for

coordination numbers 8, 6 and 3

are > 0.732 , > 0.414 and > 0.155 , respectively.

Unstable

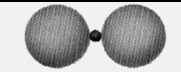
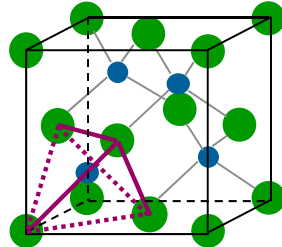

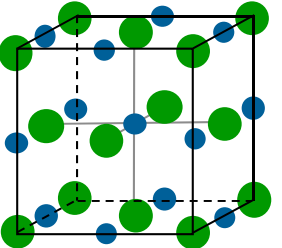
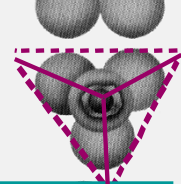
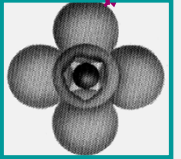
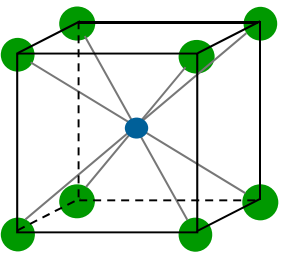
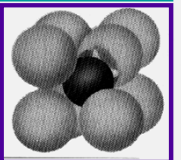


Stable

Coordination # and Ionic Radii

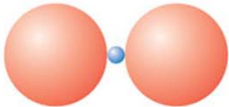
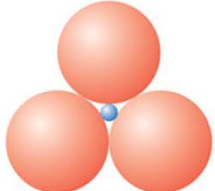
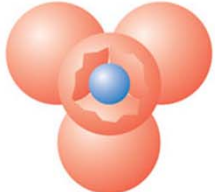
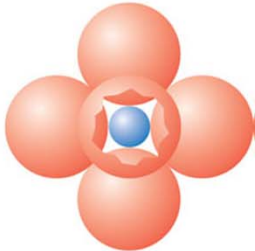
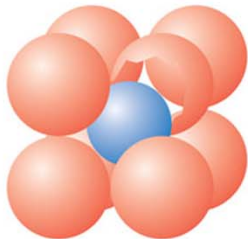
- Coordination # increases with $\frac{r_{\text{cation}}}{r_{\text{anion}}}$

To form a stable structure, how many anions can surround around a cation?

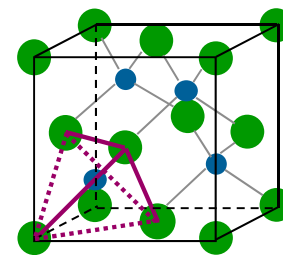
$\frac{r_{\text{cation}}}{r_{\text{anion}}}$	Coord #				
< 0.155	2	linear			ZnS (zinc blende) Adapted from Fig. 12.4, <i>Callister & Rethwisch 8e.</i>
0.155 - 0.225	3	triangular			NaCl (sodium chloride) Adapted from Fig. 12.2, <i>Callister & Rethwisch 8e.</i>
0.225 - 0.414	4	tetrahedral			
0.414 - 0.732	6	octahedral			CsCl (cesium chloride) Adapted from Fig. 12.3, <i>Callister & Rethwisch 8e.</i>
0.732 - 1.0	8	cubic			

Adapted from Table 12.2,
Callister & Rethwisch 8e.

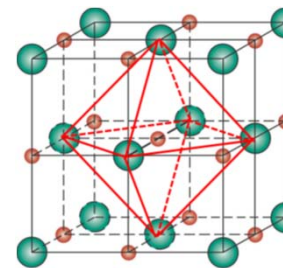
Coordination # and Ionic Radii

Coordination Number	Cation-Anion Radius Ratio	Coordination Geometry
2	< 0.155	
3	0.155–0.225	
4	0.225–0.414	
6	0.414–0.732	
8	0.732–1.0	

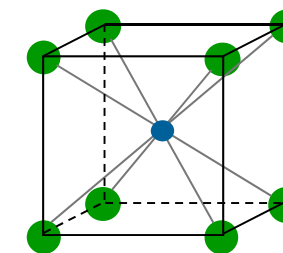
- Coordination # increases with $\frac{r_{\text{cation}}}{r_{\text{anion}}}$



ZnS
(zinc blende)

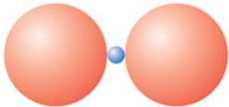
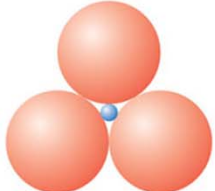
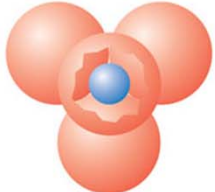
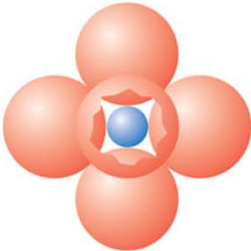
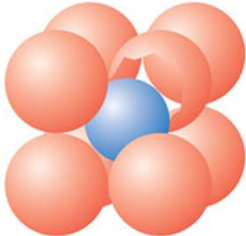


NaCl
(sodium chloride)

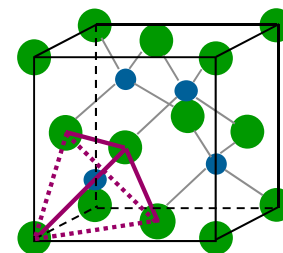


CsCl
(cesium chloride)

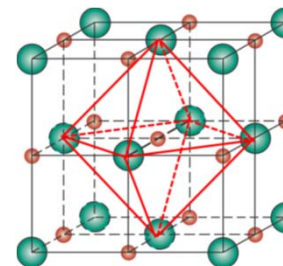
Coordination # and Ionic Radii

Coordination Number	Cation-Anion Radius Ratio	Coordination Geometry
2	< 0.155	
3	0.155–0.225	
4	0.225–0.414	
6	0.414–0.732	
8	0.732–1.0	

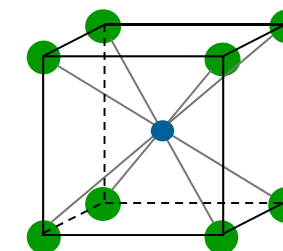
- Coordination # increases with $\frac{r_{\text{cation}}}{r_{\text{anion}}}$



ZnS
(zinc blende)



NaCl
(sodium chloride)



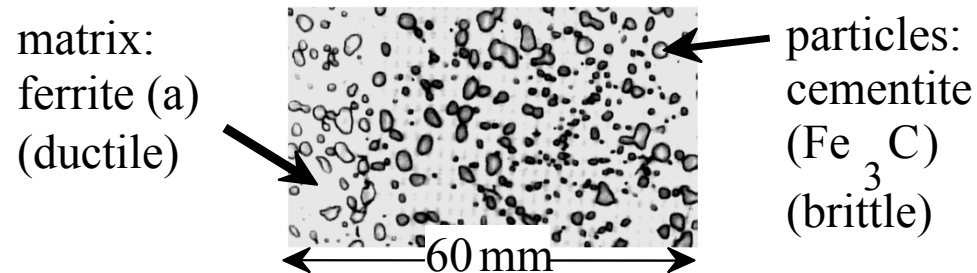
CsCl
(cesium chloride)

Composites

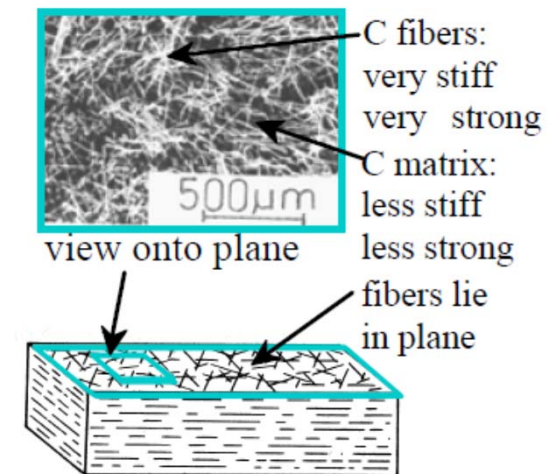
- Combination of two or more individual materials
- A composite material is a material system, a mixture or combination of two or more micro or macroconstituents that differ in form and composition and do not form a solution.
- Multiphase materials with chemically different phases and distinct interfaces
- Design goal: obtain a more desirable combination of properties (principle of combined action)
e.g., High-strength/light-weight, low cost, environmentally resistant
- Properties of composite materials can be superior to its individual components.
- Examples: Fiber reinforced plastics, concrete, asphalt, wood etc.

Composite Characteristics

- **Matrix:**
 - softer, more flexible and continuous part that surrounds the other phase.
 - transfer stress to other phases
 - protect phases from environment
- **Reinforcement (dispersed phase):**
 - stiffer, high strength part (particles or fibers are the most common).
 - enhances matrix properties

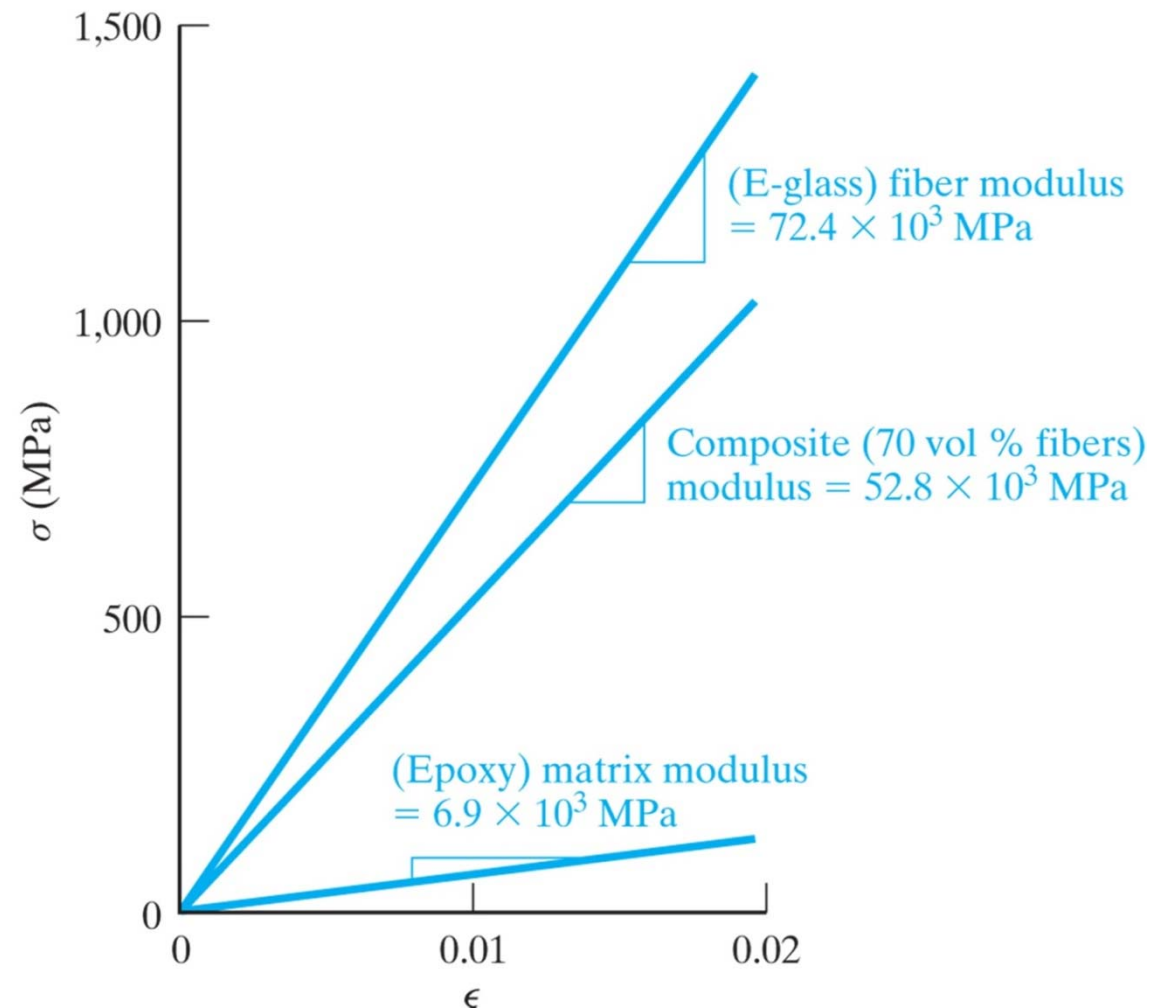


Spheroidite steel



Composite Characteristics

Simple stress–strain plots for a composite and its fiber and matrix components. The slope of each plot gives the modulus of elasticity.



Composite

Depends on:

- properties of the matrix material.
- properties of reinforcement material.
- ratio of matrix to reinforcement.
- matrix-reinforcement bonding/adhesion.
- mode of fabrication.

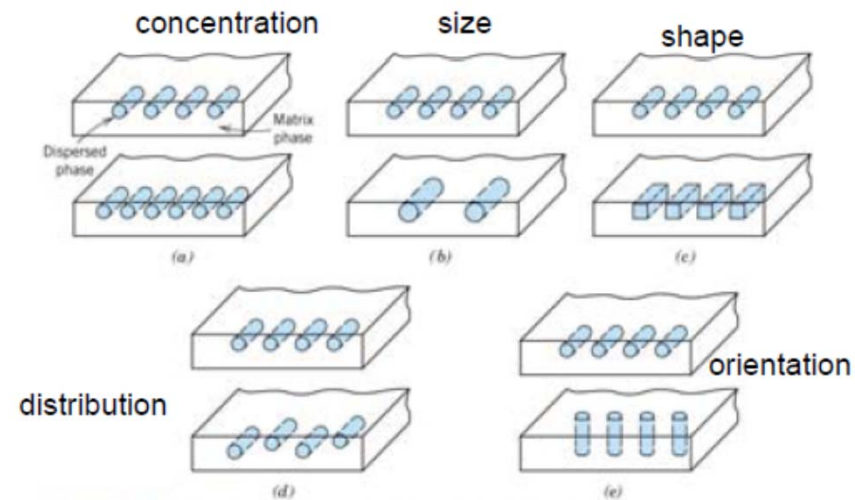
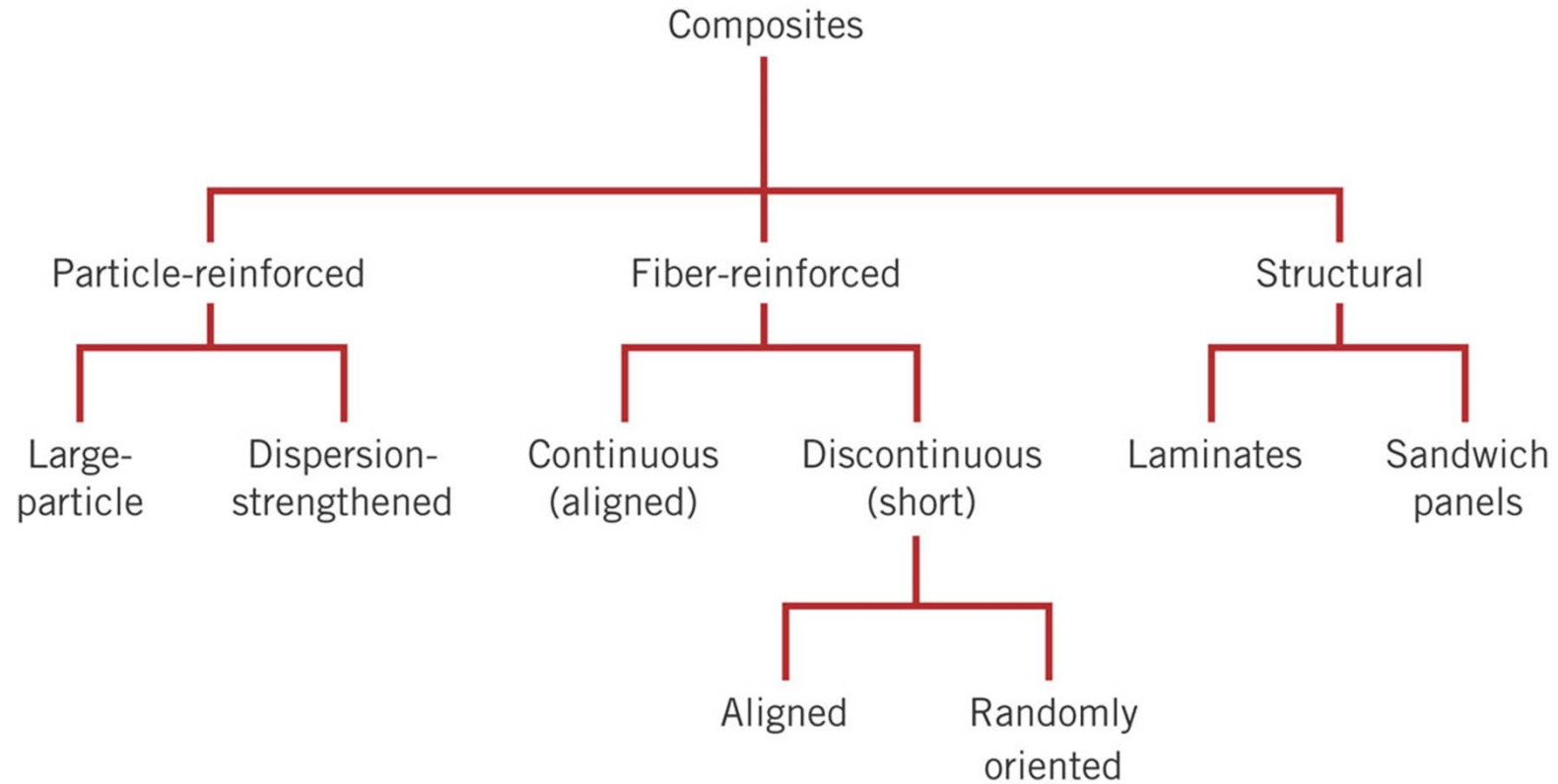


FIGURE 16.1 Schematic representations of the various geometrical and spatial characteristics of particles of the dispersed phase that may influence the properties of composites: (a) concentration, (b) size, (c) shape, (d) distribution, and (e) orientation. (From Richard A. Flinn and Paul K. Trojan, *Engineering Materials and Their Applications*, 4th edition. Copyright © 1990 by John Wiley & Sons, Inc. Adapted by permission of John Wiley & Sons, Inc.)

Classification of Composites

Reinforcement-based

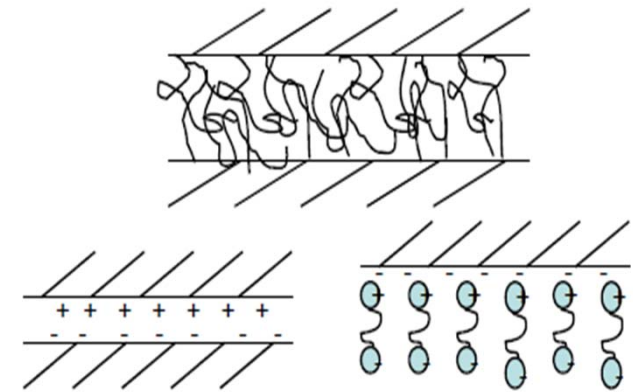


Matrix of Composites

Fiber-Matrix Interface

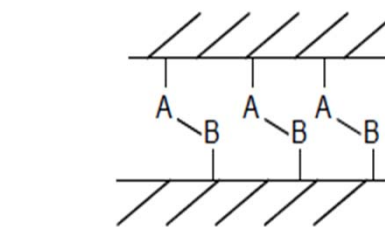
1. Molecular entanglement (interdiffusion):

- Entanglement of molecules at the interface.
- Especially important in fibers that are precoated with polymers.
- Molecular conformation/structural and chemical aspects.



2. Electrostatic attraction

- Depends on surface charge density.
- e.g. glass fibers, polymers with chargeable groups.



3. Covalent bonding

- Usually the strongest fiber matrix fiber-interaction.
- The most important in many composites.



4. Mechanical adhesion

- Interlocking of 2 rough surfaces
- e.g. thermosetting resins

Particle-Reinforced Composites

Particle-reinforced

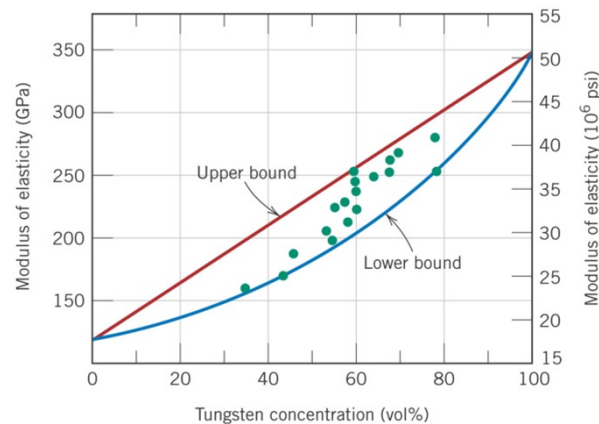
Fiber-reinforced

Structural

- **Elastic modulus, E_c , of composites:**
 - two “rule of mixture” extremes:

$$\text{upper limit: } E_c = V_m E_m + V_p E_p$$

Data:
Cu matrix
w/tungsten
particles



lower limit:

$$\frac{1}{E_c} = \frac{V_m}{E_m} + \frac{V_p}{E_p}$$

- **Application to other properties:**
 - **Electrical conductivity, σ_e :** Replace E 's in equations with σ_e 's.
 - **Thermal conductivity, k :** Replace E 's in equations with k 's.

Fiber-Reinforced Composites

Particle-reinforced

Fiber-reinforced

Structural

Fiber Phase

- Smaller diameter fiber is stronger than bulk in most materials

Whiskers

- very thin single crystals that have extremely large aspect ratios.
- high degree of crystallinity and virtually flaw free – exceptionally high strength.
- usually extremely expensive.
- some whisker materials: graphite, SiC, silicon nitride, aluminum oxide.

Fibers

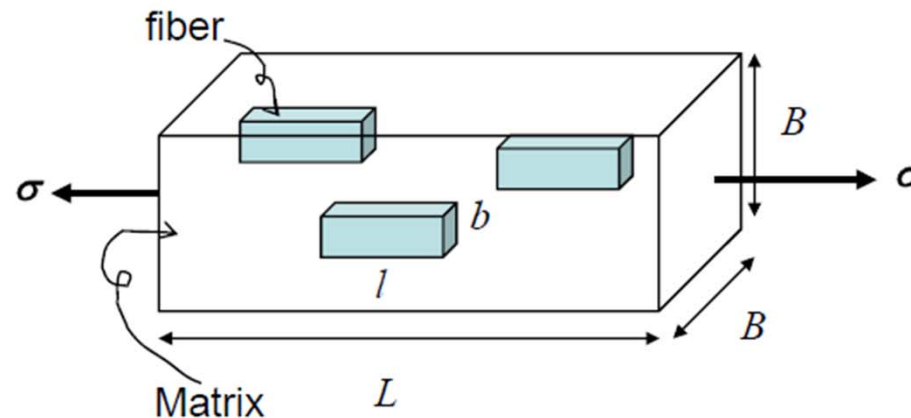
- polycrystalline or amorphous.
- typically: polymers or ceramics (polymer aramids, glass, carbon, boron, SiC...

Fine Wires

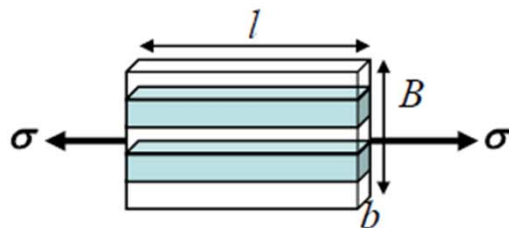
- relatively large diameter, often metal wires.
- e.g. steel, molybdenum,

Continuous-aligned fibers

Consider a composite with fibers having square rod geometry with stress in the **longitudinal direction**...



When $l \gg l_c$ (typically $l > 15 l_c$), we can simplify to:



$$\text{then } \frac{\Delta L_{\text{composite}}}{l} = \frac{\Delta L_{\text{fiber}}}{l} = \frac{\Delta L_{\text{matrix}}}{l}$$

$$\text{since } \varepsilon \equiv \frac{\Delta L}{l}, \quad \varepsilon_{\text{composite}} = \varepsilon_{\text{fiber}} = \varepsilon_{\text{matrix}}$$

Isostrain Conditions

Continuous-aligned fibers

Longitudinal loading

What about elastic modulus?

Total force on the composite is simply the sum of forces on matrix and fibers:

$$F_{composite} = F_{matrix} + F_{fiber}$$

since $\sigma = \frac{F}{A}$ or $F = \sigma A$, we have:

$$\sigma_c A_c = \sigma_m A_m + \sigma_f A_f$$

Recall stress-strain relation: $\sigma = E\varepsilon$ Sub-in for stresses...

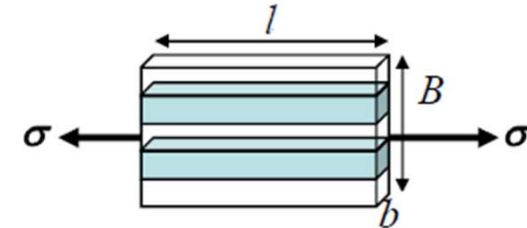
$$E_c \varepsilon_c A_c = E_m \varepsilon_m A_m + E_f \varepsilon_f A_f \xrightarrow{\text{rearrange}} E_c = \frac{E_m \varepsilon_m A_m}{\varepsilon_c A_c} + \frac{E_f \varepsilon_f A_f}{\varepsilon_c A_c}$$

Imposing isostrain condition $\varepsilon_c = \varepsilon_f = \varepsilon_m$,

$$E_c = E_m \frac{A_m}{A_c} + E_f \frac{A_f}{A_c}$$

"Area fractions"

Prefer volume fractions....



Continuous-aligned fibers

Longitudinal loading

Total area = Bb

A_f = number of fibers x area of each fiber = Nb^2

Then we have: $\frac{A_f}{A_c} = \frac{Nb}{B}$

Also, $A_m = A_c - A_f$
or

$$\frac{A_m}{A_c} = 1 - \frac{A_f}{A_c} = 1 - \frac{Nb}{B}$$

For volume fractions: $V_c = Bbl$

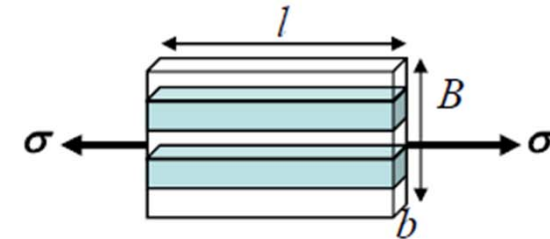
$$V_f = Nb^2l$$

Then: $v_f = \frac{V_f}{V_c} = \frac{Nb^2l}{Bbl} = \frac{Nb}{B} = \frac{A_f}{A_c}$ same as "area fraction"

Similarly, $v_m = \frac{A_m}{A_c}$

$$E_{cl} = v_m E_m + v_f E_f$$

Longitudinal elastic modulus
(upper bound for E_c)



Continuous-aligned fibers

Longitudinal loading

What about the load on the matrix and the fibers?

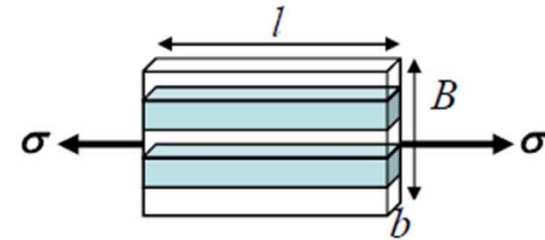
$$\frac{F_f}{F_m} = ? \quad F = \sigma A$$

$$\frac{F_f}{F_m} = \frac{\sigma_f A_f}{\sigma_m A_m} = \frac{\sigma_f v_f}{\sigma_m v_m}$$

With $\sigma = E \varepsilon$

$$\frac{F_f}{F_m} = \frac{E_f \varepsilon_f v_f}{E_m \varepsilon_m v_m} \xrightarrow{\text{Isostrain conditions}}$$

$$\frac{F_f}{F_m} = \frac{E_f v_f}{E_m v_m}$$

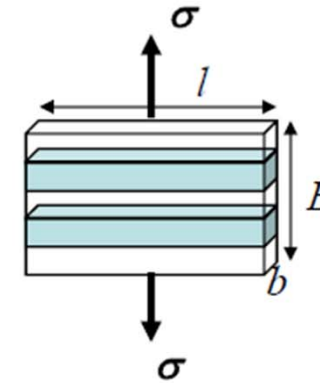


Continuous-aligned fibers

Transverse loading

Stress, rather than strain, is the same in this case:

$$\sigma_c = \sigma_f = \sigma_m \quad \text{ISOSTRESS condition}$$



The total elongation will be the sum of elongation of the components:

$$\Delta L_c = \Delta L_f + \Delta L_m$$
$$\text{Strain} = \varepsilon_c = \frac{\Delta L_c}{L_c} = \frac{\Delta L_f}{L_c} + \frac{\Delta L_m}{L_c} \quad \text{Where } L_c = \text{length of the composite along the direction of stress} = B$$

Again, to express in terms of volume fractions:

$$L_f = Nb$$
$$\frac{L_f}{L_c} = \frac{Nb}{B} = v_f \quad \leftarrow$$
$$\text{Rearrange } L_c = \frac{L_f}{v_f}$$
$$\text{Similarly, } L_c = \frac{L_m}{v_m}$$

Continuous-aligned fibers

Transverse loading

$$\begin{aligned}\varepsilon_c &= \frac{\Delta L_c}{L_c} = \frac{\Delta L_f}{L_c} + \frac{\Delta L_m}{L_c} \\ &= \frac{\Delta L_f}{L_f / v_f} + \frac{\Delta L_m}{L_m / v_m} = v_f \left(\frac{\Delta L_f}{L_f} \right) + v_m \left(\frac{\Delta L_m}{L_m} \right)\end{aligned}$$

ε_f

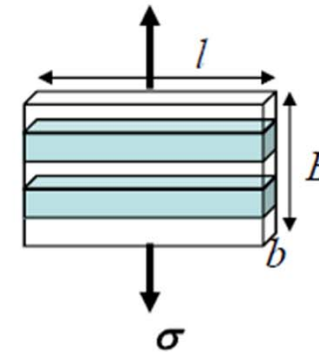
ε_m

$$\varepsilon_c = v_m \varepsilon_m + v_f \varepsilon_f$$

From stress-strain relation; $\varepsilon = \frac{\sigma}{E}$

$$\frac{\sigma_c}{E_c} = v_m \frac{\sigma_m}{E_m} + v_f \frac{\sigma_f}{E_f}$$

Applying isostress condition and rearranging gives



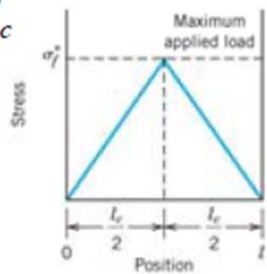
**Lower bound for E_c
(transverse modulus)**

$$E_{ct} = \frac{E_m E_f}{v_m E_f + v_f E_m}$$

Discontinuous & aligned fibers

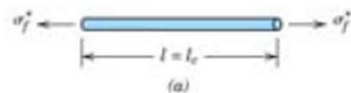
Discontinuous when fiber length $< 15 l_c$

If $l > l_c$

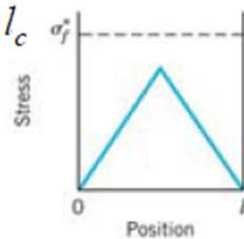


$$\sigma_{cd}^* = \sigma_f^* v_f \left(1 - \frac{l_c}{2l}\right) + \sigma_m' v_m$$

Similar to continuous case, except for the length factor.



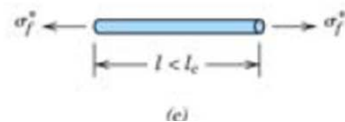
If $l < l_c$



$$\sigma_{cd'}^* = \frac{l \tau_c}{d} v_f + \sigma_m' v_m$$

$\tau_c =$ Smaller of:

- fiber-matrix bond strength
- matrix shear yield strength.



Fiber-Reinforced Composites

Discontinuous & random fibers

Discontinuous, random 2D fibers

- Example: Carbon-Carbon
 - process: fiber/pitch, then burn out at up to 2500C.
 - uses: disk brakes, gas turbine exhaust flaps, nose cones.

$$E_c = E_m V_m + K E_f V_f$$

efficiency factor

(depends on v_f and E_f/E_m):

- aligned 1D: $K = 1$ (anisotropic)
- random 2D: $K = 3/8$ (2D isotropy)
- random 3D: $K = 1/5$ (3D isotropy)

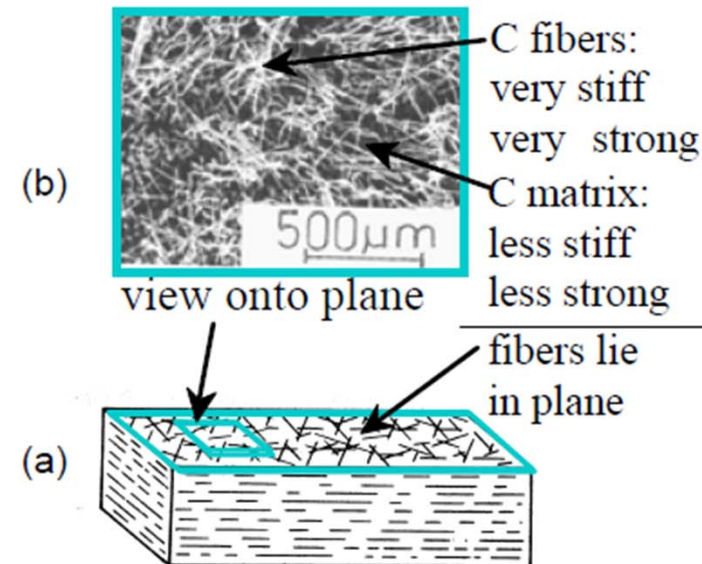


Table 16.2 Properties of Unreinforced and Reinforced Polycarbonates with Randomly Oriented Glass Fibers

Property	Unreinforced	Fiber Reinforcement (vol%)		
		20	30	40
Specific gravity	1.19–1.22	1.35	1.43	1.52
Tensile strength [MPa (ksi)]	59–62 (8.5–9.0)	110 (16)	131 (19)	159 (23)
Modulus of elasticity [GPa (10^6 psi)]	2.24–2.345 (0.325–0.340)	5.93 (0.86)	8.62 (1.25)	11.6 (1.68)
Elongation (%)	90–115	4–6	3–5	3–5
Impact strength, notched Izod (lb _f /in.)	12–16	2.0	2.0	2.5

Source: Adapted from Materials Engineering's *Materials Selector*, copyright © Penton/IPC.